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ABSTRACT

The proceedings of a conference that discussed the universities' responsibilities in the preparation of mathematics teachers are presented. The report covers a series of meetings at Queen's University, Kingston, Canada, from August 31 to September 3, 1977. The conference was restricted to university mathematicians and mathematics educators in Canada, and was designed to be a starting point for more professional contact and public discussion of mathematics education. Major portions of the report contain papers on: Mathematics Education Research in Canada, Innovations in Teacher Education Programs, Objectives of Mathematics Education, Reports from Five Working Groups, and Reflections After the Conference. Appendices contain a list of conference participants, working group discussion papers, a conference bibliography, and a national survey of Canadian Teacher Education Programs. (MP)

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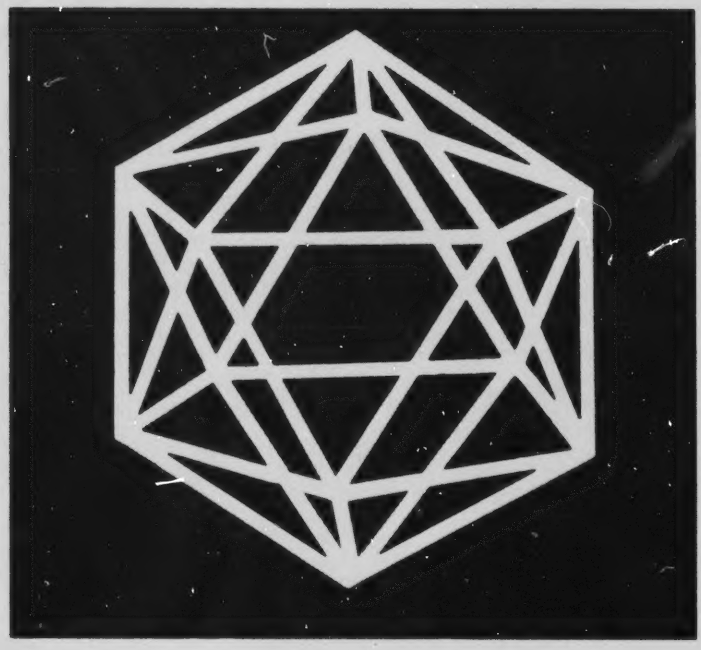

EDUCATING TEACHERS OF MATHEMATICS: THE UNIVERSITIES' RESPONSIBILITY

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Proceedings of a Conference sponsored by the
Science Council of Canada at Queen's University

EDUCATING TEACHERS OF MATHEMATICS:
THE UNIVERSITIES' RESPONSIBILITY

Edited by

A. J. Coleman, W. C. Higginson, and D. H. Wheeler

Science Council of Canada
1978 04

PREFACE

The decision to organize a conference to discuss the universities' responsibilities in the preparation of mathematics teachers sprang from two related desires. One was to achieve some discussion of the issues concerning mathematics education raised in Mathematical Sciences in Canada;[1] the other was to bring together a group of mathematicians and mathematics educators across Canada to explore the possibility of improving inter-provincial contact and communication.

Although many of the people consulted in the preparation of the Back-Ground Study had a great deal to say about mathematics education in Canada, and particularly about its shortcomings, this aspect of the report itself has received very little public discussion. One of the contributory reasons may be the lack of a national organization with any responsibility to consider and speak about mathematics education in Canada. Although there are a number of provincial associations of mathematics teachers, the only professional organization with a national membership is the National Council of Teachers of Mathematics, and this is understandably more concerned to speak for mathematics education in the United States where the bulk of its membership resides.

A small conference seemed more likely to achieve the initial contact and communication that we wanted, so we decided to restrict the conference membership to university mathematicians and mathematics educators. The subject of teacher preparation immediately suggested itself as the appropriate part of mathematics education to focus on. We drew up a programme and an invitation list for a meeting at Queen's University, Kingston, from August 31st to September 3rd, 1977. The Science Council of Canada generously agreed to sponsor the conference and meet the expense.

The report that follows covers most of what can be reported of the conference proceedings, and it is published as a contribution to the national discussion of mathematics education in Canada. The conference was short, the participants had to get to know each other, and many of the discussions that took place did not lend themselves to being written-up in detail, so the final report should be seen as an indication of the issues that were discussed, not a definitive statement on them.

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[1] K. P. Beltzner, A. J. Coleman, and G. D. Edwards, Mathematical Sciences in Canada, Background Study No. 37. Ottawa, Ontario: Science Council of Canada, July, 1976.

We were cheered beforehand by the ready acceptance by most of the people who were invited, and by their assurances afterwards that the conference had been worthwhile. Seen as a first step in the direction of more professional contact and more public discussion, we think the Kingston conference has a future.

We are indebted to the Science Council of Canada for financial assistance, to conference participants for their enthusiastic response, particularly to Speakers and Working Group Chairmen and reporters, and to Noreen Mills, Tom Racey, Patricia Whitaker and Eileen Wight for their unstinting, high quality technical support.

A. J. Coleman
W. C. Higginson
D. H. Wheeler

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Invited Papers

MATHEMATICS EDUCATION RESEARCH IN CANADA:
A PROSPECTIVE VIEW

T. E. Kieren

1. Introduction

1.1 What is our venue?

Mathematics education research, like much of educational research, has not been given an entirely charitable construction in the past. Its value has been questioned, and even when it contained solid advice regarding theory as well as practice, this advice was ignored in favour of the fad of the moment or the comfort of old ways in the face of the problems of the day.

Still, the document Mathematical Sciences in Canada cites a general dissatisfaction with mathematics programmes and instruction in the schools and universities and other tertiary institutions as well. There is a strong call for improvement of programmes and practices. What might be the bases of this improvement? At least some of the input for these bases should come from sound educational research.

Mathematics education research makes use of mathematical ideas, but certainly differs from research in pure mathematics both in method and content. The issues of concern for a mathematics educator - for example, "How does a learner build up the idea of function?" - may be of little interest to the mathematics research community (although it could be argued that real insight into mathematics *per se* comes from studying its learning). Similarly, psychological researchers, although sometimes using mathematical settings, are not generally interested in the mathematical development of an individual or the psychological aspects of mathematics acquisition or use. Thus, the researcher in mathematics education has a unique sphere of interest: the development of mathematical constructs in persons, the mechanisms used in this development, and the conditions necessary for this development.

1.2 Necessity for connectedness

The unique sphere described above is not one with closed or smooth boundaries. Because the problems of studying mathematical constructs and their growth and development is complex, this research must be internally and externally connected. These external connections might be with mathematical or psychological research. But it is as likely that they will be with a broader spectrum involving other areas of endeavour, such as research on learning of science or the development of higher level constructs, or general research on teaching. The complexity of problems facing the mathematics education researcher suggests that single isolated studies will yield very limited results, hence internal connectedness and cooperative efforts are needed. Perhaps the critical comments referred to above stem from the lack of such connectedness in much previous mathematics education research.

1.3 Overview

It is the purpose of this paper to develop a picture of the potential for mathematics education research in Canada. Although the next section of the paper attempts to give snapshots of past and current mathematics education research, the thrust of the paper will be prospective and not retrospective. To give a framework to a general research scheme, Section 3 will deal in some detail with the notion of a "construct" and the ways in which constructs grow and are developed by human learners. Suggestions for major types of research efforts as well as suggestions for mechanisms for fostering such research in a Canadian context are found in Section 4.

2. The status of mathematics education research

2.1 Cognitivist vs. behaviourist

Over the past fifteen to twenty years, research in mathematics education has been influenced by one of the two sides in a more general conflict in educational thinking. One side, the behaviourists, have sought immutable cause-and-effect laws relating the sequencing of instructional stimuli and predictable student responses. This camp has sought to develop instructional sequences individualized on the basis of the learner's current learning history and has made use of hierarchies of behaviourally-stated objectives. The cognitivist camp has sought to discover the schema which individuals have and use in dealing with their environment. They are interested in an individual's development over time and the tailoring of instructional settings congruent with the learner's stage of development and mental structures.

These educational positions (greatly oversimplified) are but a recent manifestation of an age-old philosophical controversy. This controversy revolves around the question, "To what extent is a human a being who simply responds to the environment for his own or the general good?" This question has been central in the fields of ethics, religion, and science, as well as in education. While it is doubtful that this question will ever be resolved, in the sense of a consensus position, it is almost certain to continue to influence the search for knowledge about human endeavours. It is certainly true that mathematics education research has moved beyond the behaviourist-cognitivist dichotomy suggested above. Nonetheless, the question of the nature of human behaviour continues to influence research and does influence the suggested course of this paper.

2.2 Trends in research

In what way does mathematics education research fit within or go beyond the dichotomy discussed above? Bauersfeld (1976) suggests a number of types of research which have recently been done, some of which transcend the specific behaviourist-cognitivist conflict and some of which represent a departure from traditional experimental methodology.

There is still an immense number of studies done using the experimental paradigm of comparing the effects of two (or more) treatments or states on mathematical achievement or affective variables. Some of these have taken into account interaction effects which can give hints for matching treatments and groups of students (Bauersfeld, p. 5), but even these have very limited contributions to make to knowledge. This is due to the complexity of the teaching-learning environment, which can easily conceal or distort experimental effects.

A second style of research is typified by the Soviet practice of "teaching experiments". Here mathematics learning is studied in a group or class over an extended period of time through variation of conditions of instruction. There is less emphasis on psychometric measures, and outcomes are reported more in terms of dynamic process descriptions (Bauersfeld, p. 6).

A third trend is seen in research which deliberately involves teachers as co-investigators. Such research studies the decision-making efforts of teachers and effects on the teaching-learning environment. Much of such research is very informal and introspective, but some has involved sophisticated study of teacher-student interaction, though there has been very little on student-student interaction.

A fourth trend, which represents a clear transcendence of the cognitivist-behaviourist polarization, is the increasing number of studies involving an information-processing approach. Here there is an attempt to describe internal mental functioning and yet to give a time sequence of actions to describe processes.

A fifth trend (and one of which this conference, and particularly this paper, is both a symptom and a part) is a search for frames of reference for knowledge about mathematics learning and development. This involves a search for statements about the nature of the mathematical sciences, about models for teaching and learning, about the nature of mathematical abilities and the interaction of these with learning environments. These more philosophical studies have been followed, particularly in the latter case, by numerous attempts to identify and trace the abilities of students across time and situations.

2.3 A note on complexity

One of the conclusions drawn from a consideration of Bauersfeld's (1976) trends, is that mathematics learning is being viewed as a more complex phenomenon and there is a movement away from research questions, paradigms, and methodologies which ignore, mask, or try to oversimplify the situation. Indeed two hypothesized theorems pertinent to this conference might be:

Complexity Theorem (C.T.):

$$C(\text{learning}) \longrightarrow C(\text{instruction})$$

Teacher Education Coroliary

$$\text{C.T.} \longrightarrow C(\text{teacher education})$$

The complexity of the task and its attendant richness are heightened as one moves from a narrow frame of reference for mathematics to a broader construct of mathematical science and its position and interconnections.

2.4 Canadian concerns

A central concern of Beltzner *et al.* (1976) with respect to mathematics education in Canada is growth. This is seen in its personal sense in the call for an education in and the opportunity to practise "mathematization". In a collective sense this growth emphasis appears in a desire for a more extensive view of mathematics, and particularly in a renewed emphasis on applications of mathematics.

This new emphasis on growth is, perhaps, a call for the renewing of and broadening of contact between the mathematics and mathematics education communities and the societal and personal dimensions of the broader Canadian community. From an "employment" point of view there is a simultaneous need for persons skilled in technology and for persons able to fill diverse service positions. Because of unique Canadian problems in communications, transportation, and resource management, Canadian solutions to these problems may be prototypic for general human problems in these areas. Because these demands are non-trivial there should be a sense of mission in the mathematics education community. Because of the technico-mathematical components of society, the goals of personal growth in mathematics should enhance the acuity with which a person can view the contemporary Canadian scene.

These growth goals call for changes in the mathematics curriculum at all levels. These changes cannot easily be incorporated within the framework of a textbook and have broad implications for teacher education as well.

Of course the above changes suggest many changes in mathematics education research. Among these is the need for a deeper and broader understanding of mathematical notions in persons of all age levels, and the patterns of growth of such notions. Beyond this broad research need, one specific area of study is the impact of computational technology on both the curriculum and learning in the mathematical sciences.

Evaluation is currently being carried out on the effects of current practice on mathematics achievement. This work, of large scale, and ongoing in many provinces, seeks to answer diverse questions. It can, should, and does, serve as a stimulus to mathematics education research.

The mathematics education research community in Canada does not have a long history or tradition. However, the recent work of this community has relevance for the concerns expressed in Mathematical Sciences in Canada as well as forming a basis for the work still to be done that is described in the remainder of this paper. A portion of this work falls directly in the category of variable relationships noted above, the particular merits and relevance of which must be judged in each individual case.

There has been substantial research work and writing in Canada on cycles within mathematics learning. These have focused particularly on the variety of personal activity and related curriculum experiences involved in what Beltzner *et al.* (1976) would call "mathematizing".

There has also been considerable recent research on cognitive development as it affects and effects mathematical development. This work has, in part, derived from the work of Piaget in content and in method. It has also been concerned with differences in structural learning across various ages.

A fourth category of Canadian mathematics education research has concerned itself with the structure, style, and manner of mathematical knowing. Some of this work has been philosophical in nature and sought either to describe aspects of personal mathematical knowing or the curriculum antecedents generative of such a process. Other work has entailed the detailed observation of persons, particularly young children, as they worked within situations with mathematical content. This work has sought to define the character of mathematical knowing as seen in the patterns of behaviour of children.

As suggested above, much of this work is closely related to the concern for personal growth. This research has gone on in a milieu of a great deal of curricular experimentation, some of which, at least, has been creative and carefully studied. This aspect of Canadian mathematics education research, informal though it may be, cannot be ignored and indeed needs strengthening.

3. On constructs

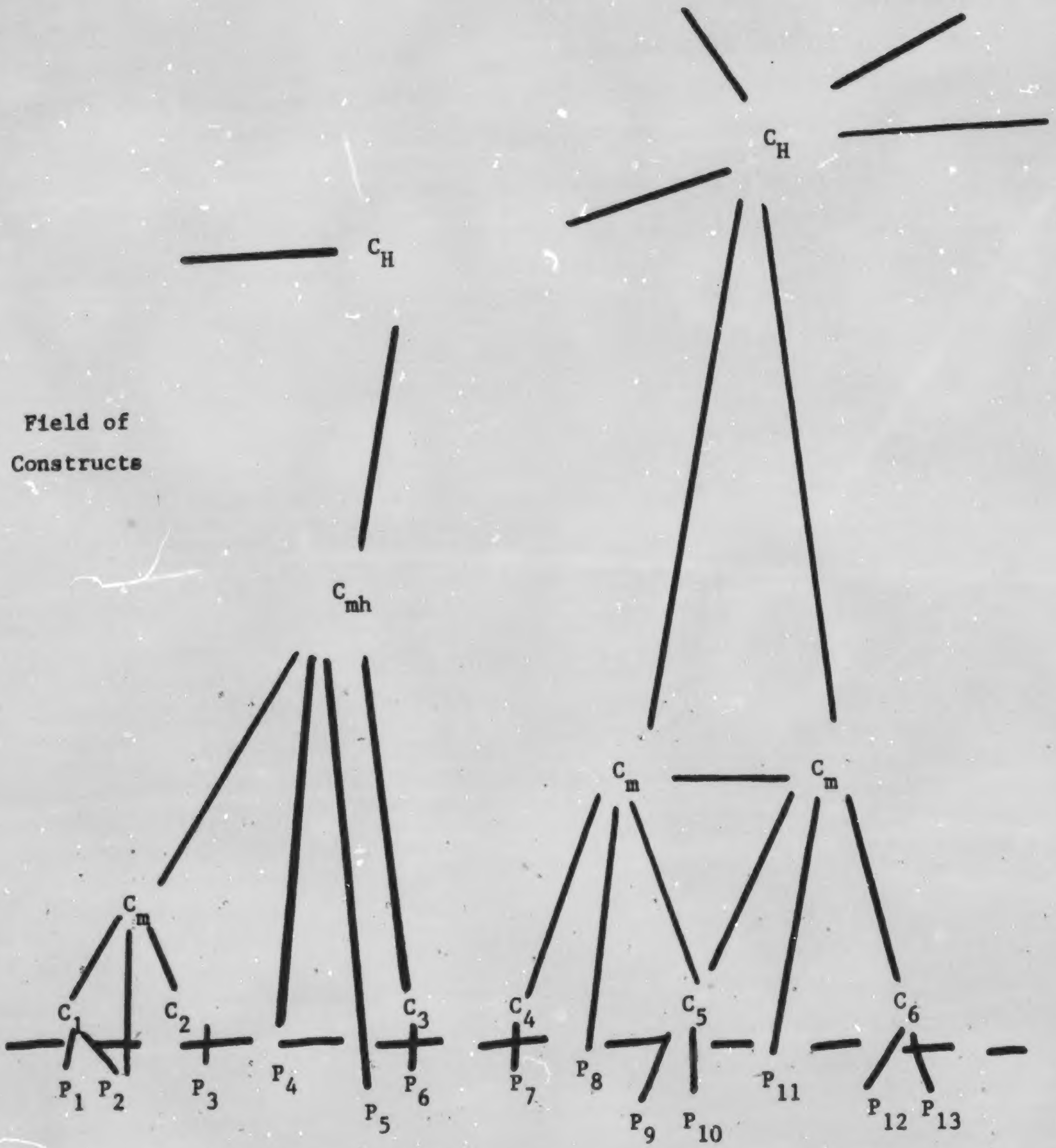
Important questions raised by Beltzner *et al.* (1976) are:

- What is the contemporary view of the mathematical sciences?
- What is a personal view of mathematics in general, and of one's own mathematics?
- How does one build up mathematical notions?
- How does one use mathematical notions? How does this use affect society?

Before proceeding to discuss possible research directions, this section of the paper gives a general characterization of mathematical knowing and the ways in which this is developed.

3.1 Margenau's idea of construct

In trying to characterize scientific epistemology, the philosopher Margenau (1961) divides phenomena into two categories. The first of these comprises the elements of physical reality, facts, or, as he chooses to term them, "protocols". These are seen as phenomena which are not dependent upon



C_H
C_H
Field of Constructs
C_{mh}
C_m
C_m
C_m
C_1
C_2
C_3
C_4
C_5
C_6
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
P_{10}
P_{11}
P_{12}
P_{13}
Plane of Protocols

FIGURE 1

human construction. The second category contains "constructs", the deliberate ideas which a person builds up about phenomena and which he or she can ultimately test against other constructs or the plane of protocols. It should be noted from Figure 1 that some constructs are in close proximity to the P-plane and offer limited explanatory power and control. Other constructs are more "abstract" - that is, further from the P-plane; these can be more powerful and give the person broader control.

A person's total mathematical construct consists of his or her network of sub-constructs, some very narrow, others much broader in their perspective. While it is difficult from this point of view to speak of the construct of mathematics or, even more difficult, the mathematical sciences, these notions result in part from societal consensus but more from the product of a combination of tests and mathematical argument.

3.2 Important characteristics of constructs

Margenau (1961) describes a number of important characteristics of constructs, two of which are especially useful for the purposes of this paper and for mathematics education in general. The first of these has been alluded to above and is termed the extensibility of the construct. This refers to the breadth or variety of phenomena to which the construct addresses itself. It has been suggested, for example, that the rote learning of computation leads to constructs which have very little power or breadth of applicability. One might say that a goal of the modern mathematics movement has been to broaden a person's constructs through the understanding of mathematical structure.

Particular mathematical constructs do not and should not stand in isolation from one another. Further, they should not stand in isolation from a person's broad range of constructs of reality. Thus constructs which are connected are of particular value. This connection may be internal or external. For example, the sub-construct of additive inverse is internally connected to the other notions about the domain of integers in a variety of ways. It is externally connected to the construct of inverse transformation in the geometric sense, and to a broader and more extensive notion of inverse in general.

Thus it can be conceived that mathematics education has a professional responsibility to provide experiences which are generative of extensive and connected mathematical constructs in our clients, our students.

3.3 On construct formation and building

3.31 On cycles

As suggested, a prominent theme of Canadian mathematics educators has been the description and study of cycles. Dienes (1961), for example, uses cycles of "play" to describe the building up of mathematical ideas - a movement from object or element play to symbolic play and hence to applicational or extensive play which may, in turn, be a foundation to a new cycle. Dawson (1971) uses the epistemology of Popper and Lakatos for a base and defines viable cycles of observation, testing, and proving (e.g., O T P, P O T) in the development of mathematical ideas. Sigurdson (1976) sees six phases in problem solving (or construct development). These are: the perception of the mathematical content of a situation; the posing of an answerable mathematical question; the making of a model or theorem to help answer the question; the validating of the theorem; the generalizing of the theorem; and, finally, perceiving and/or developing the axiomatic supports for the model or theorem. While probably not unique to mathematical construct development, all three accounts describe formalizing and generalizing processes which are part of the mathematical milieu.

The cycles described above might be termed micro-cycles in that they pertain to the development of a single subconstruct or the solution to a single problem. However, they are suggestive of a cycle of macroscopic construct development which may be pertinent to larger mathematical constructs. As seen in Figure 2, this cycle has three general stages. In the first, the person encounters a construct in a variety of representations and particularly explores the elements of its mathematical variates. While representation theorems in mathematics are designed to produce logical economy through isomorphisms, it may not make constructive or peda-logical sense to subsume construct development under a single variate.

The second stage of the cycle involves formal development. This involves the ability to work with the construct quickly and easily using standard forms, notations, etc.

The third level entails advanced exploration using the construct as a basis or tool. This may involve more advanced mathematics (e.g. rational numbers ⟶ rational expressions) or may involve some special technical application (rational approximation to the internal circumference of a metal tube).

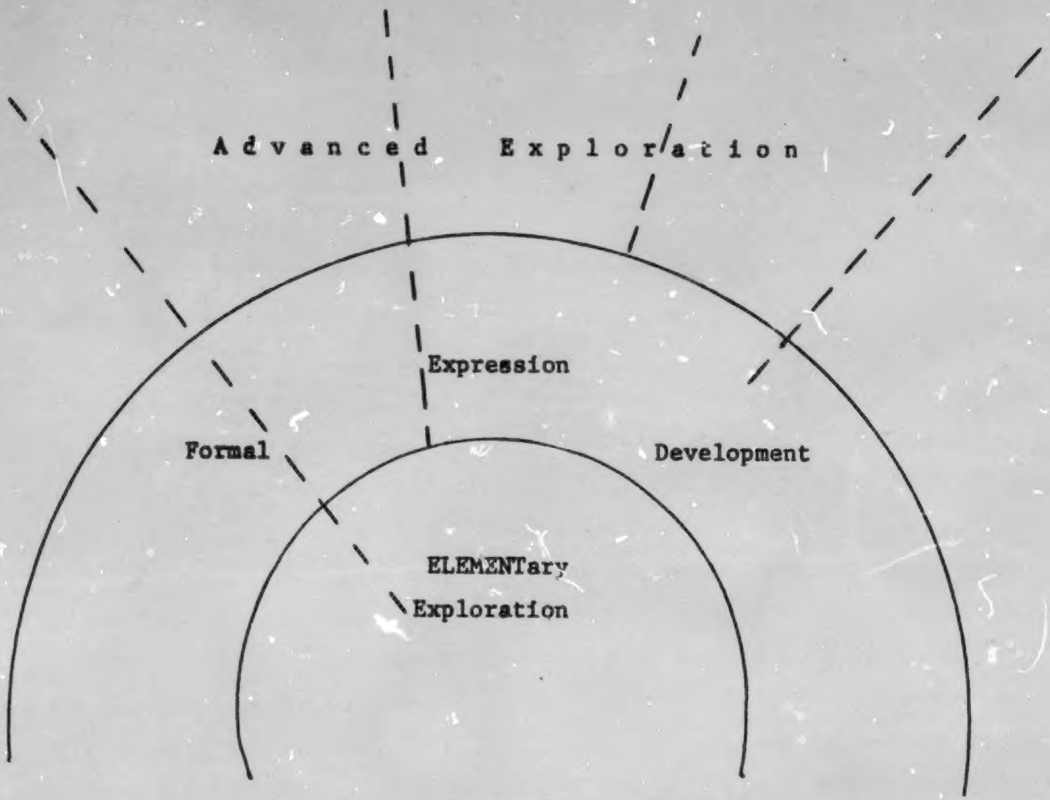


FIGURE 2

This development cycle can be applied to a variety of mathematical constructs. In school mathematics, particularly at the upper elementary and secondary school levels, we have concentrated on the second level to the detriment of the complete cycle of construct development and the consequent broadening of the scope of an individual's view of mathematics.

3.32 On a "types" problem

One of the effects of this almost exclusive concentration on the formal development level of construct formation is the accompanying view of a construct entirely as a behavioural surface of formal manipulation, frequently computation. While not denying the importance of such manipulation, we notice that this has led to the formation of empty or sterile constructs. Margenau (1961) saw a similar problem in a science which emphasized experimentalism without supporting theory, the results of which were shallow and subject to collapse. Similarly, empty mathematical constructs collapse, as seen in poor personal performance in later mathematics or in its application.

In a way this collapse suggests an analogy in the instruction-learning field to the classic Russellian theory of types. In that theory confusion of types led to paradoxes. In construct development, and curricular experiences designed to that end, the confusion of a formal surface with a complete construct leads to meaninglessness for the learner (Olson, 1977).

3.33 On mechanisms

How are mathematical constructs built up by children and adults? This still remains a puzzling question which should be a focus for research. It is apparent that a person, consciously or unconsciously, uses a variety of mechanisms and schemes in exploring, developing, and using mathematical constructs. One category of such mechanisms, *developmental mechanisms*, although partly the product of experience are not the product of any formal learning experience and are not dependent upon such experience. Examples of such are conservation of various sorts, class inclusion, and proportionality. The second category of mechanisms, *constructive mechanisms*, although general and in a sense "natural", are likely to be the product of some type of instruction. Examples are counting, partitioning, and algorithmic thinking. Such mechanisms deserve much more detailed study and their potency needs to be recognized in our curriculum-making efforts at all levels.

Building Mechanisms

Developmental	Constructive
Conservation	Counting
Simultaneous Comparison	Partitioning
Reversibility	Applying Structure
INRC group	Using Inequalities
	Transformations

FIGURE 3

3.4 On "what is meant" and "what is learnt"

Bauersfeld (1976) claims that there are important distinctions to be made between what is meant, taught, and learnt. It has been the purpose of this rather extended section to characterize in general "what is meant" in mathematics, using Margenau's notion of construct. This notion entails something built up by individuals in their own minds. Thus there can be at least a rough parallel between what is meant and learnt provided there is an appropriate form of mathematical analysis, that the individual's construct is not a behavioural surface without support, and that curriculum and instruction are based on broad mathematical constructs.

4. Implications and directions for research

The discussion in the previous section contains a wide variety of researchable hypotheses. Bauersfeld's (1976) excellent analysis suggests a spectrum of potential for mathematics education research. The suggestions given in this section are by no means the "whole cloth" of research. Yet they represent a rather broad but hopefully cohesive direction and dimension of research. Further, this research has obvious links to much successful personal work already ongoing in Canada. Further, it can have some direct if not immediate (and maybe this possibility is underestimated) results for mathematics learners at all levels.

4.1 Basic constructs

Some of the needed curriculum research is analytic and philosophical in nature. Given today's world, what are the basic constructs to be included in a mathematics curriculum? This question was asked in a very limited way by the reform movement of the last 25 years, but the answers seemed to dwell more on the depth of the constructs than on the kinds of construct. The question was also answered in a speculative way for a limited range of students in the Cambridge report (1963), and the work at CEMREL which has followed from it. There have also been curricula (e.g. Papy (1970)), or parts of curricula (e.g., geometry in Ontario) which reflect certain definite answers to this kind of question.

At the early elementary school level and perhaps in university honours curricula there is less need for this kind of study. However, in upper elementary school, secondary school, many university programmes (e.g., teacher education), and in other tertiary education programmes, answers to such basic construct questions are overdue.

In answering such questions, the nature of society will have to be considered. For example, in what ways does the availability of computing devices enter into deliberations on basic constructs? Similarly the nature and content of mathematics and the basic knowledge about human development *vis-à-vis* mathematics will also be bases for answering such construct questions.

4.11 Basic mechanisms

A question similar to the above can be asked about mechanisms. Within our selected constructs what are the mechanisms useful for their development? Are there mechanisms which have a broad range of functioning (e.g., counting) and deserve a central curriculum role of their own?

While such a question calls for philosophical and psychological analysis, it also calls for active research with persons at various age levels. This research will entail the observations of persons in situations designed to "trigger" the particular mechanism and would attempt to ascertain how the mechanism functioned and developed.

4.12 Construct validation

There are many ways in which a construct can be tested. One way is to lay its sub-constructs against the qualities of a maturely functioning person within the domain of consideration and see analytically (and empirically if this is desired) if the construct-based curriculum meets functional needs.

A second validation is to test whether the developed construct is generative of learning activities appropriate to the group of intended learners. Further, do such activities also induce the development of constructs in a vast majority of the intended learners? (This has been a serious "textbook" problem in the past.)

4.13 On extensibility and connectedness

One important test of a developed curriculum is an assessment of its constructs and mechanisms. To what extent does the curriculum highlight powerful constructs and mechanisms? (This has been partly done in past searches for "unifying" mathematical concepts.) If a curriculum is to be useful today, it must be based on constructs of broad importance which enlarge the scope of the learner's exploratory and explanatory power.

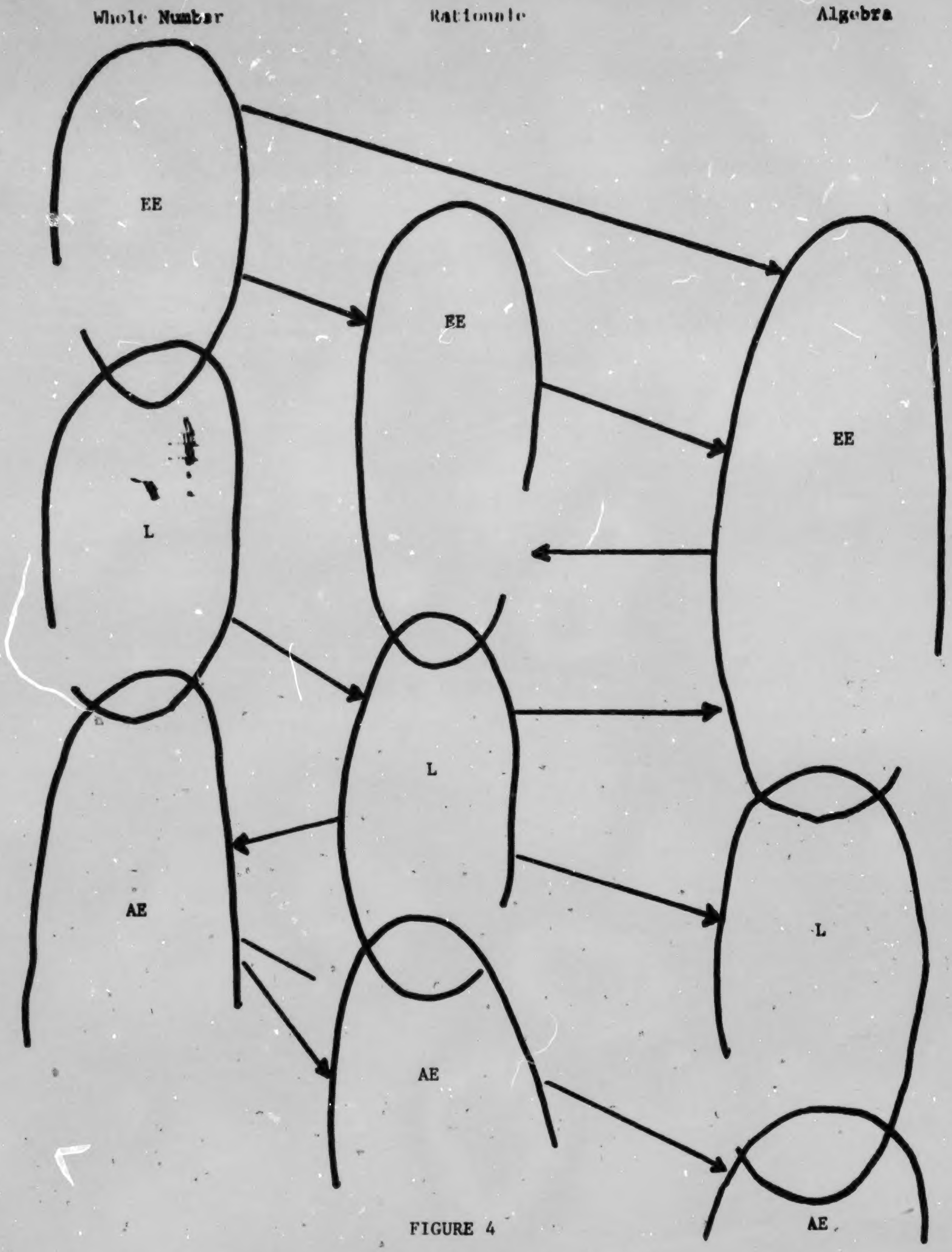


FIGURE 4

4.2 Cycle research

The notion of "cycle" has been important in this paper and in recent Canadian mathematics education research. There are a variety of researchable questions which fall in this category.

For discussion purposes, this paper has posed a three-level construct development cycle. Given a particular basic construct, what are the characteristics of each of the three levels or stages? In some senses this is a very "nitty-gritty" question. Yet it is central to the development of learning experiences. If there is no answer to such a question, the foregoing philosophical discussion remains only that. Although answering this question has an "armchair" component, it should also have a large component of work with appropriate learners in particular experiences.

A related question pertains to mechanisms. Which mechanisms contribute to development at which stages? Answering this question allows a different way of studying the validity and particularly the extensibility of particular mechanisms.

A third category of "cycle" questions concerns micro-cycles. How are the cycles suggested by Dawson (1971), Dienes (1961), or Sigurdson (1976) pertinent to construct and/or mechanism development at particular macro-cycle stages? For example, are they more pertinent at the two exploratory levels than at the formal development level? Are they (the micro-cycles) different in character at various macro-cycle levels? Are they developable and are they unique to each construct or mechanism? These questions present a rich field for study both at an experience development and experience testing level. They, with other questions in this section, allow researchers, and indeed force researchers, to be precise about their intents, transactions, and outcomes - to use Stake's (1971) terms.

4.21 On technology

Technology, particularly computing technology, will have a profound effect on mathematics learning and instruction as it affects and becomes part of the basic constructs. The kinds of activities which relate to the use of such technology will also have an impact on instruction. Four such activities are:

- algorithm design
- coding
- machine application
- data organization and study.

One might consider the first of these to be representative of a profound mechanism. Engel (1976) suggests that the mathematics curriculum centre on this basic mechanism.

A less controversial matter is suggested by the figure below.

	E.E.	F.D.	A.E.
Algorithm Design			
Coding			
Machine Application			
Data Study			

FIGURE 5

How do these informatic activities contribute to levels of construct development? Because of the computational power provided by a machine, some areas of advanced exploration of mathematical constructs become feasible and convenient. It may be that algorithm design and coding are key personal activities in the formal development aspect of construct building. Of course, these statements are but two of many testable hypotheses.

4.22 On language cycles

A major concern in mathematics learning is the use of language and the formality of this language. A question with respect to construct development is whether there is a language-use cycle which parallels the development cycle. One hypothesized cycle is given below.

Informal codes

Formal language

Technical language

FIGURE 6

The first level is suggestive of learner-developed expressions about the mathematical phenomena being explored. There may be different codes pertaining to different variates of a construct, for example.

The second level relates to the standard language used with a construct. Learning such language may well present a connotation problem with a single standard code now applying to a wide variety of construct variates (Hillel, 1976).

The third level pertains to certain "standardized" uses of language which are peculiar to an application of a construct (e.g., rational numbers applied to measuring devices in a millwright's trade). Here the user must relate this language to both the standard language and his or her construct. This proposed cycle and its relationship to construct development contains numerous testable hypotheses for researchers and developers.

4.3 Mechanism research

There has been considerable research to date in the area of developmental mechanisms. There is considerable Canadian research on the growth of such mechanisms with respect to mathematics (Harrison, 1976; Drost, 1977). There has been some (Bourgeois, 1976), but much less, attention paid to the growth of constructive mechanisms (counting, partitioning, algorithm design). This area of study needs considerable research with attention paid to the choice of mechanisms and to the development of useful measuring devices and techniques.

A second question is the relationship between mechanism growth and construct development. This has been studied on a limited basis - for example counting and whole numbers (Steffe, 1976), and measurement and fractional numbers (Owens, 1977; Babcock, 1977). There are many important questions yet to be asked in this area.

4.4 Brain physiology

Research on brain-functioning is just reaching a stage where it can have impact on mathematics education research. How is brain-functioning a basis for construct and mechanism growth and use? Questions of this nature will likely prove an interesting field of basic research in the near future.

4.5 A note on teacher education

Research activity such as that suggested above has implications for teacher education. Some of these are direct in the sense that they concern the necessary mathematical constructs and mechanisms for teachers. Perhaps more important is to think about mathematics learning in terms of the learner's constructs and mechanisms. For the teacher of younger children, this likely means a more intensive mathematical education than is currently acquired in Canadian teacher education programs. For other teachers this likely means a broadening of their education in significant ways, both in terms of applications to science, commerce, social science, etc., and of extensible constructs and mechanisms.

4.6 Summary

The research problems suggested above are far from being clean and simple. They represent a recognition of the complexity of mathematics and its learning. In general, solutions to these problems will give explanatory assistance to those dealing with mathematics learning in the raw, the teachers, but certainly do not offer a panacea for currently perceived ills in our field.

By design and by necessity the research problems suggested above are interconnected. It is only by a network of research that the complex problems posed can be studied effectively.

Beyond the research already suggested, and included in it, is a need for studying learning relationships in mathematics. What is the nature and impact of teacher-student and student-student interaction with respect to the learning cycles, and to construct growth and use by individuals?

There is a need for much more interrelated mathematics education research to tackle these problems. Perhaps our small numbers in Canada and our personal interrelationships will allow us to engage in such interrelated research.

4.7 Recommendations

What can be done to effect the cooperation needed in Canadian mathematics education research? In the short run two things suggest themselves. Since we need better information as a base, it would be useful to have a bibliography. briefly annotated, of work done in the last five years. This should include university and school-sponsored research and should include various graduate-level theses, as well as research done by professionals in the field. Such a bibliography would outline our current strengths, weaknesses, and personal resources, *vis-à-vis* the task suggested above. It would also give some indication of potentials for cooperative effort.

An active newsletter describing current work and supporting interpersonal research communication is a second short-term need. This would be a specialized informal document and should complement the more formal organs already available.

In the longer run there is a great need for cooperative research. Because the problems are complex, several persons are needed to investigate parts of these problems in a pre-planned way using a language which is understandable to all working in an area. At a first level such cooperation needs to occur among professional mathematics education researchers. But because the problems have many facets and levels, this cooperation needs to include the broader academic community, including linguistics experts and philosophers, for example, as well as mathematicians, computer scientists, and psychologists.

Howson (1976) states that an increasing number of teachers are active in curriculum development on a world wide basis. There is need for cooperation among researchers and teachers (who could be the same persons). The former can give the latter advice about the framework and parameters of the curriculum. The teachers can provide dynamic feedback about various situations to the researcher.

Finally, there is a need for groups of researchers and teachers to meet regularly on problems in mathematics education in Canada. Because of our geography, it may be well to look at the French IREM as a model of regional groups and centres. It would be hoped that such centres would provide the support and life necessary to tackle the problems outlined above in a substantial and ultimately practical way.

References

Babcock, G. "Measurement Ability and Rational Number Achievement," a research proposal, University of Alberta, Edmonton, 1976.

Bauersfeld, H. "Research Related to the Mathematical Learning Process," a report for ICME III, Karlsruhe, F.R.D., August, 1976.

Beltzner, K., Coleman, A.J. and Edwards, Gordon D. Mathematical Sciences in Canada. Science Council Background Study No. 37. Ottawa: Science Council of Canada, 1976.

Bourgeois, R. "Young Children's Behaviour on Division Problems," a Ph.D. dissertation, University of Alberta, Edmonton, 1976.

Dawson, A.J. "A Fallibilistic Model for Instruction," Journal of Structural Learning, Vol. 3, No. 1, 1971.

Dienes, Z.P. Building Up Maths. London: Hutchinson Press, 1961.

Drost, D. "Geometric Sectioning Ability and Geometry Achievement," a Ph.D. dissertation, University of Alberta, Edmonton, (Fall) 1977.

Engel, A. "The Computer and Mathematics," a paper presented at ICME III, Karlsruhe, F.R.D., August, 1976.

Harrison, D.B., Personal Communication, University of Calgary, Calgary, 1976.

Hillel, J., Personal Communication, Concordia University, Montreal, 1976.

Howson, A.G. "A Critical Analysis of Curriculum Development in Mathematics Education," in Programme II, ICME III, Karlsruhe, F.R.D., August, 1976.

Margenau, H. Open Vistas. New Haven: Yale University Press, 1961.

Olson, A.T., Personal Communication, University of Alberta, Edmonton, 1977.

Owens, D. "The Relationship of Area Measurement and Learning Initial Fraction Concepts by Children in Grades Three and Four," Faculty of Education, University of British Columbia, Vancouver, 1977.

Papy, G. Modern Mathematics I. London: Collier-Macmillan, 1968.

Report of the Cambridge Conference. Boston: Houghton Mifflin, 1963.

Sigurdson, S.E., Personal Communication, University of Alberta, Edmonton, 1976.

Stake, R. "The Countenance of Educational Evaluation," in Teachers College Record, 68:523-40, April 1967.

Steffe, L. "Quantitative Comparisons as a Readiness Variable for Arithmetical Content Involving Rational Counting," part of a final report of a study conducted for PMDC, Georgia Center for the Study of Learning and Teaching Mathematics, 1976.

INNOVATIONS IN TEACHER EDUCATION PROGRAMMES

C. Gaulin

First, I wish to thank the organizers of this conference for inviting me to give this lecture. I also wish to congratulate them for having succeeded in organizing such a meeting of mathematics educators and mathematicians from all over Canada and for having done it so well. Such an opportunity to gather, to share information, to discuss current problems and issues in mathematics education, and to plan concerted activities for the future has long been badly needed in Canada and I sincerely hope follow-up activities will be organized on a more permanent basis.

Like Tom Kieren, I shall attempt to focus on general trends and issues in the field of teacher education, more particularly on those which might be relevant in Canada in the near future. I must confess that my knowledge of the present state of teacher education in many Canadian provinces is deficient, and I apologize in advance for possibly omitting to mention important realizations, concerns, or trends in some parts of the country.

My presentation will follow the following lines:

1. The traditional organization of teacher training
    - A. Preservice teacher training (PRESET)
    - B. Inservice teacher training (INSET)
2. Some innovations and new directions developing in teacher training

    A study by Coutts and Clarke on the future of teacher training in Canada
3. One current programme showing several innovations in in-service teacher training: the PERMAMA programme
4. Some thoughts about the role and responsibility of universities in teacher education in the future.

Although many remarks will relate to teacher education in general, it remains understood throughout this presentation that the innovations and trends reported here specifically concern the training of teachers of mathematics.

1. The traditional organization of teacher training

In order to better appreciate some of the new directions developing in teacher training which will be mentioned in Part 2, I shall first briefly point out some features of the traditional organization of teacher training which is still quite common today.

1.A PRESERVICE TEACHER TRAINING (PRESET)

1.A.1 Traditionally teacher education in universities has been chiefly conceived and organized in terms of preservice teacher training. In most places, inservice teacher training has been subordinated to PRESET or treated as a second-order priority.

1.A.2 Generally PRESET in Canada is done in the universities. Prospective elementary teachers are trained as generalists within a B.A./B.Ed. programme, with little mathematics and some mathematics methodology. On the other hand, prospective secondary teachers are trained as specialists within a B.Sc./B.Ed. programme, with a great deal of mathematics and some mathematics methodology.

1.A.3 In most universities, programmes of preservice teacher education have an extensive common core of compulsory courses or activities and do not allow much opportunity for flexible individually-tailored programmes for students. The philosophy underlying this is essentially that PRESET should prepare every teacher for his whole career and accordingly should include many courses and activities considered to be fundamental and essential.

1.A.4 To a large extent, decisions about the structure of PRESET programmes and about the objectives of the courses they include are made *a priori* by government and university people, with little participation by the students concerned and/or by people actually teaching in schools. Moreover little continuous evaluation of such programmes is usually done.

1.A.5 Traditionally, except for the practice teaching period and a few workshop-style activities, a majority of the courses are of the lecture type, followed by exercises, assignments, etc. This seems true for both mathematics courses and foundations courses in education.

1.A.6 There are at least three classical problems in preservice teacher education which still persist and deserve special mention:

1.A.6.a Lack of integration of the various components of PRESET programmes

Many PRESET programmes, whether aiming at preparing elementary school generalists or secondary school specialists, look like mere juxtapositions of many components for which a heterogenous group of people is responsible. Any kind of genuine integration seems to be missing, even between the components in education or between many of the courses offered in mathematics. No wonder, then that so many criticisms are heard about the way teachers are trained, since "the learner himself is expected to integrate in his learning all the knowledge his teachers were not able to integrate in their teaching: a high expectation, a vain expectation," as Hans Freudenthal so properly pointed out during the Pécs conference last month.

1.A.6.b Lack of balance and the gap between theory and practice in PRESET

Many criticisms are still heard about many PRESET programmes being too theoretical. Specific reference is often made, for example, to the inadequacy of the practice teaching component, and to some courses (in education, in mathematics, or in mathematics education) whose objectives and methodology are too remote from the needs and concerns of a school mathematics teacher. Bridging the theory-practice gap remains very difficult because of well-rooted attitudes among university people, such as the "first you learn it, then you apply it" axiom, or the belief expressed by Boileau's classical, *"Ce que l'on conçoit bien s'énonce clairement et les mots pour le dire arrivent aisément"*. Another difficulty arises from the fact that many teacher trainers, including mathematics educators or mathematicians, have themselves little knowledge of what is actually going on in the schools. (In the United States, competency-based teacher education is gaining popularity, but I am afraid that, considering the way it is implemented in some colleges, it is likely to go too far the other way and put too much emphasis on practice.)

1.A.6.c Lack of coordination between the "mathematics" and "mathematics education" components of PRESET programmes

In many PRESET programmes, more particularly in those for secondary school mathematics teachers, a big gap still exists between courses in mathematics and courses in mathematics education (often called "methods courses" although they usually cover much more than teaching methods). There are several reasons for this--for example both types of courses are not often given concurrently. But the main reason seems to be a lack of communication and cooperation between mathematicians and mathematics educators, representing two groups of professional people with different specializations, basic concerns, and types of activities, and which in addition are often located in different places. To improve the situation, some mathematics departments offer a few mathematics courses especially devised for prospective teachers. However, many recommendations made by mathematicians about the mathematical training of a would-be teacher overemphasize content and disregard any related didactical problems, under the implicit assumption that "first you learn mathematics, then you (eventually) learn to teach it!" Of course, such an attitude does not help to bridge the existing gap.

1.A.7 In the mathematics courses which are part of PRESET programmes, most of the teaching continues to be product-oriented, with little explicit emphasis on processes characteristic of mathematical activity (e.g., mathematization, heuristics, etc.).

1.B INSERVICE TEACHER TRAINING (INSET)

1.B.1 During the past fifteen years, a great number and diversity of INSET courses and activities have been organized, partly by universities and partly by other organizations: school boards, teacher associations, ministries of education, and even private organizations. Some have been credit-bound, while many others have not. Some have been university-based, while many others have rather been school-based (e.g., off-campus university courses or professional development days). Such INSET activities often give participating teachers the opportunity to eventually obtain an increase in salary.

I.B.2 During the last fifteen years, the majority of inservice teacher training courses and activities in mathematics have been of the updating type, and to some extent of the RE-training type. During the "new math revolution" of the sixties, for example, most practising teachers had to be literally re-trained in terms of the content and the methodology which were characteristic of the "new mathematics" curricula and textbooks. More recently, many INSET courses and workshops have had to be organized to prepare inservice teachers for the implementation of the *Système International* (SI) in the schools.

1.B.3 In recent years, there has been a growing awareness among university people that inservice teachers constitute a clientele with specific needs and expectations, background and experience, attitudes etc. Accordingly, it is now more widely accepted that in many respects INSET should be conceived and organized differently from PRESET, with much more flexible entrance requirements (e.g. analogous to those of the Open University in England). Many university professors, however, are still reluctant to accede to this and fear that the consequence might well be an unacceptable lowering of academic standards (in the traditional sense).

1.B.4 The majority of INSET courses and activities, particularly those offered or sponsored by universities, are not part of the professional task of the teachers concerned.

1.B.5 In the mathematical component of INSET programmes, most of the teaching continues to be product-oriented, with little explicit emphasis on processes characteristic of mathematical activity (e.g. mathematization, heuristics, etc.).

2. Some innovations and new directions developing in teacher training

During recent years, interesting innovations have been tried in teacher training, more particularly in North America, in England, and in the Scandinavian countries. Such innovations reflect some general medium term or long-term directions at present developing in PRESET and INSET in many countries. I shall now attempt to sketch such current trends.

2.1 "CONTINUING EDUCATION" AS A CONCEPTUAL FRAMEWORK FOR PRESET AND INSET

The general concept of "continuing education" (*"education permanente"*) as applicable to the education of every individual throughout his or her life, is quite fascinating, but it still needs clarification and more agreement about its meaning and ways to make it operational enough. There are strong indications, however, that this concept can advantageously serve as a conceptual framework for PRESET and INSET.

In broad terms, in the case of any individual, education may be viewed as a life-long process of which the development includes the following phases:

(a) basic general education, acquired in school (compulsory schooling period) as well as outside school;

(b) education in preparation for a career, which may be acquired in various ways and places (this includes further general education as well as professional preparation and apprenticeship);

(c) further education during a career, which may also be acquired in various ways and places (this includes general as well as professional further education or training).

Of course this should be refined with "loops" (to account for changes in career orientation), and with a provision for skipping phase (b) in some cases, but I shall stick to the above rough model for the purpose of the discussion here.

In the particular case of a school teacher, PRESET in universities is clearly part of phase (b), while INSET is part of phase (c). It must be clearly kept in mind, however, that phases (b) and (c) include not only activities related to the professional task of a teacher, but also other kinds of activities which may be educational in a general sense and contribute to the personal development of an individual (e.g. getting information or experience in other subjects or occupations through personal study or involvement).

In my opinion, one of the features of a genuine concept of continuing education is that the above phases (a), (b), and (c) are not merely juxtaposed, but are conceptually and practically interlocked through deliberate planning and action.

Using continuing education as a conceptual framework for PRESET and INSET therefore implies in particular that:

(i) PRESET and INSET are conceptually inseparable, complementary parts of a continuous process, with many interdependent components.

An immediate consequence of this is that PRESET should no longer be thought of nor organized as if it were aimed at preparing a teacher for a whole career. If properly organized, INSET should allow any in-service teacher to eventually take any course he may have "missed" and which was optional in PRESET. This in turn implies that in pre-service teacher training the common core of compulsory courses and activities for all students should be reduced to a minimum, in order to allow more opportunities for flexible individual programmes (see 1.A.3 above). Considerable efforts should then be made to integrate at least the components of the compulsory common core in PRESET through concerted work of teams of university specialists (see 1.A.6.a above).

Another consequence is that INSET should be viewed primarily as a continuation of PRESET throughout the career of a teacher, allowing him or her to discuss problems actually met in the classroom or in the school, to learn more about fundamental relevant subjects, to share experiences or initiate concerted action with other teachers, to get up-to-date information about current trends, new teaching methods and media, etc. From this point of view, INSET should include, but not be focused chiefly on RE-training activities that may be made necessary by sudden, often carelessly planned changes in curricula or textbooks or teaching methods, or by a significant change in career orientation (see 1.B.2 above). Moreover INSET should include a much greater range of activities which are relevant and worthwhile for teachers: courses, workshops, discussion periods, projects, participation in professional conferences, participation in a research, etc. (Of course this is not easy in a university credit-bound INSET programme. However, the more INSET becomes part of the professional task of the teacher in the future, the more such variety may be possible.)

(ii) Neverthless, preservice teachers and inservice teachers constitute different clienteles, each with its specific needs and expectations, background and experience, attitudes, etc. Accordingly the objectives and methodology of many INSET activities and the overall structure of INSET programmes offered in universities are likely to be very different from those in PRESET (see 1.B.3 above).

In my opinion, there is a long-term trend slowly emerging in the direction I have just sketched, particularly where training teachers of mathematics is concerned. I feel, however, that there is still a long way to go before a genuine concept of "continuing education" becomes clear and operational enough in our universities (even if many already offer so-called "continuing education courses"!).

2.2 MORE DELIBERATE COOPERATION IN PRESET AND INSET BETWEEN UNIVERSITIES, SCHOOLS, AND PEOPLE TAKING PART IN TEACHER TRAINING ACTIVITIES

A medium-term trend which is increasingly noticeable is towards a more deliberate cooperation in PRESET and INSET between three groups: universities; school representatives; people taking part in preservice and inservice teacher training activities.

On the one hand, this means that student-teachers in PRESET and inservice teachers in INSET are likely to play an every-increasing role in decision-making concerning teacher training: for example, as members of programme committees or as active participants in surveys conducted about their needs, expectations, and evaluations of current teacher training activities (see 1.A.4 above).

On the other hand, there are already strong indications that a much closer cooperation will be established in the near future between universities and school representatives in the organization of teacher training, particularly along the following lines:

(i) In PRESET, more efficient practice teaching or internship schemes allowing more deliberate interaction between certain theoretical courses and classroom experiences, and accordingly narrowing the classical gap between theory and practice (see 1.A.6.b above).

(ii) School-focused INSET with increased responsibility for school boards and schools and with new roles played by universities. (An international "Conference on Strategies for School-Focused Support Structures for Teachers in Change and Innovation" was held in Stockholm in October 1976, sponsored by O.E.C.D., and follow-up international conferences have already been planned for November 1977 and 1978.)

(iii) Creation of many local "professional centres" or "teachers' centres" (school- or school board-based preferably) serving many purposes, but considered chiefly as privileged places for a variety of INSET activities. (In the U.S.A. there already exist many such centres of different types, and in England the "James Report", published in 1972, recommended the creation of a country-wide network of "professional centres", although many such centres have existed for years.) Universities might contribute in many ways to the realization of INSET activities in such locally-run "teachers' centres" and in particular play non-traditional roles in the organization of INSET activities which may not be credit-bound but which will be part of the professional task of the teachers concerned. Moreover, it might well be a worthwhile idea for universities to plan and realize a few PRESET activities in close cooperation with "professional centres" in their area.

The above medium-term directions are particularly noticeable as far as training teachers in mathematics is concerned.

2.3 OTHER POSSIBLE DIRECTIONS IN PRESET AND INSET IN MATHEMATICS

I would like to point out two new directions which might develop in teacher training. They however remain more problematical than the preceding trend because they presuppose significant changes in deeply-rooted traditions and attitudes among university people--in particular among mathematicians and mathematics educators. These two new possible directions apply to both PRESET and INSET.

The first one is a greater emphasis on processes characteristic of mathematical activity (e.g. mathematization, heuristics, etc.) both in mathematics courses and in mathematics education courses, which are still much too product-oriented. This is certainly highly desirable in both preservice and inservice courses, particularly in today's climate where mathematics curricula tend to be biased by an excessive emphasis on specific behavioral content objectives (see 1.A.7 and 1.B.5 above).

The second one is a significant change in the way research in mathematics education is viewed, planned, and conducted. In my opinion, much more research and development in this field should be planned and conducted in cooperation with practising teachers as part of INSET activities. Schemes for training preservice teachers might also allow the involvement of more (undergraduate) prospective teachers in some research projects. I feel there should be a strong interdependence between (1) the evolution of mathematics curricula and teaching methods and media in schools; (2) inservice training activities in mathematics; (3) much of the research and development done in mathematics education which is not of the fundamental type.

A study by Coutts and Clarke on the future of teacher training in Canada

Let us pause to look at some of the conclusions of a study conducted by H.T. Coutts and S.C.T. Clarke on "The future of teacher education" and presented at the American Educational Research Association Convention in New York City, February 1971.

According to this study, a sample of chief administrators of the English-speaking teacher education institutions in Canada estimated that teacher education in the foreseeable future (1975-1980) would move in the following directions:

1. Teacher education would be centred around an extended internship.
2. Teacher education would continue throughout the teacher's career, with frequent use being made of sabbatical leave for one or two semesters to be spent at a university.
3. Candidates for teacher education, both for admission to preparatory programmes and for first certification, would be required to exhibit a satisfactory standard of excellence in: speech, English usage, mental health, and human relations.
4. Teachers would be prepared more intensively as subject specialists.
5. Although there would be a common core of learning for all, each candidate's programme would be individually tailored.
6. The common core learning required by all teachers would include:
   (a) preparation in working as a member and as a leader of a group or team which might be a mixture of superordinates and subordinates, or persons all at one professional level;
   (b) a great deal of attention to ethics, morals, attitude development, and character formation;
   (c) preparation in the use of the latest education technology and media.
7. Teacher education would be about half "common core" for all candidates and about half specific to specialization in terms of: function, level, and staff discrimination.
8. Teacher education would emphasize the process of learning (observing, clarifying, inferring, inquiring, reasoning, remembering) as contrasted with the product (information, knowledge, concepts, generalizations).

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3. One current programme showing several innovations in inservice teacher training: the PERMAMA programme.

In order to illustrate some innovations which are taking place in INSET at present, I would like to sketch one particular INSET programme which I have been associated with and which is increasingly popular in Québec. It is called the "PERMAMA programme". where PERMAMA stands for *"PERfectionnement des MAîtres en MAthématique"*.

I shall give here only a very general description of the programme and anyone interested in more specific information may consult a few papers on the subject or contact the Director of PERMAMA.

3.1 Preliminary remarks

(a) PERMAMA is an inservice teacher training programme run by *Télé-université du Québec* (a branch of the Université du Québec). Most collaborators work in Montréal.

(b) PERMAMA is a programme primarily intended for high school mathematics teachers from all over the Québec territory.

(c) At present PERMAMA is a credit-bound programme leading to a "certificate" and eventually to a bachelor's degree. To be admitted to the programme, one must have taught in schools for at least three years.

(d) About 1300 teachers are currently registered in the programme.

(e) PERMAMA started in 1972. It has been built upon the experience and the understructure which have grown out of a previous Government-run inservice training programme for high school mathematics teachers (1966-1971), called "C.R.P.M." (*Cours de Recyclage et de Perfectionnement en Mathématique)*. As a matter of fact, since 1966, the philosophy and the type of organization of inservice training of secondary mathematics teachers have gone through three distinct phases in Québec, giving rise to three types of INSET programmes: (1) C.R.P.M. from 1966 to 1971; (2) PERMAMA 1st generation, from 1972 to 1975; PERMAMA 2nd generation, since 1975.

(f) Teachers registered in the PERMAMA programme participate in courses and activities in their leisure time. This type of inservice teacher training is currently not part of the teacher's professional task.

3.2 A network of teachers' centres for INSET activities in mathematics

Teachers registered in the PERMAMA programme participate in courses and activities in local or regional teachers' centres. There are, at this time, 97 such PERMAMA centres spread over the Québec territory. In each centre there is one so-called "*moniteur-animateur*", generally a mathematics teacher himself; his job consists mainly in doing some organization and in serving as an "animator" during the working and discussion periods of "permamists" (N.B. his role is not to teach!). Periodically all *moniteurs-animateurs* meet in order to share their experiences, to prepare for new PERMAMA courses and coming activities, to give feedback about recent courses and activities offered, and to participate in decision-making about the continuation of the programme.

Remark: *moniteurs-animateurs* are paid a salary for their work.

3.3 A bank of "modules" of various types allowing personalized INSET programmes

Up to now a bank of about 60 "modules" has been established. Each module is a learning unit, generally using various media, provided with a guide for the *moniteur-animateur*. Supposing a group of teachers using a module meets once a week on the average, and that each member does required work at home or in schools every week, then finishing the module may require at least four to eight weeks. Modularization of previous "courses" (offered in the PERMAMA 1st generation programme) has been very successful and has given much more flexibility to PERMAMA.

At the present time five types of modules may be found in the bank:

(a) modules focused on mathematical content (elementary algebra, geometry, algebraic structures, statistics, vectors, Boolean algebras graph theory, derivative, programming, integral, number systems, etc.)

(b) modules focused on mathematical activity (problem solving, mathematization, etc.)

(c) modules focused on didactical problems (concept learning in mathematics, student-teacher relations, learning through problem solving, teaching geometry, laboratory activities in mathematics, using worksheets for teaching mathematics, etc.)

(d) modules focused on the realization of "projects" in schools: after identifying a problem or a need in mathematics teaching in their schools, a group of teachers think of a relevant "project" to realize, plan it carefully, realize it, and evaluate the results (N.B. supervision is provided by the PERMAMA *"équipe-pédagogique"* for such projects)

(e) modules permitting teachers to plan personalized sequences of PERMAMA modules with appropriate information and cooperation ("management modules").

To some extent teachers registering in the PERMAMA programme have the opportunity of tailoring personalized sequences of modules. Only three modules are compulsory. There are, however, a few constraints which may limit such an individualization of INSET programmes. For example, some modules are offered only if a minimum number of teachers want to use it simultaneously (because group work and discussion are considered essential to make such modules profitable) and accordingly negotiation and cooperation between teachers in the same PERMAMA centre may be necessary to find an optimal compromise.

During 1977, 400 teachers who were registered in the PERMAMA programme initiated 141 "projects" between January and August. Such projects may play a tremendously dynamic role in promoting better teaching of mathematics in schools and in making more interdependent: (1) research and development in mathematics education; (2) in-service teacher training; (3) the evolution and the improvement of mathematics teaching in schools.

PERMAMA full-time staff includes the *équipe pédagogique*, consisting of a few mathematicians and mathematics educators and many experienced high school mathematics teachers. This group mostly works on the preparation and testing of modules, keeping close contact with schools and with the *moniteurs-animateurs* of the 97 PERMAMA centres spread over Quebec. A few other full-time people are more specifically concerned with management so that the network of PERMAMA local centres functions properly and that information about PERMAMA activities is disseminated appropriately.

3.4 Participation of teachers in decision-making and in the organization

Participation of the teachers concerned in the PERMAMA programme is insured in various ways. For example, many experienced high school mathematics teachers are part of the *équipe pédagogique*. On the other hand, periodic surveys are made in order to determine the degree of satisfaction of the "permamists" in regard to existing modules, their suggestions for improvement, and their desires concerning the production of new modules. Many teachers also cooperate in pre-experiments with modules in preparation, or make a more systematic evaluation of existing modules. Finally, the group of all "permamists" has a representative in the *Comité directeur* of the programme.

During 1976-1977, registrations in the PERMAMA programme have increased by 80%, showing the degree of satisfaction of inservice teachers with respect to this second generation programme. It is obviously much more flexible, relevant, and stimulating than the previous (first generation) PERMAMA programme which was much more uniform and too exclusively content-oriented.

4. Some thoughts about the role and responsibility of universities in teacher education in the future

I have already sketched a few possible medium-term and long-term directions developing in teacher education, and made allusions to changes they might imply as far as the role and responsibility of universities in PRESET and INSET is concerned. I do not wish to add much more to that. Let me therefore finish with four short remarks:

4.1 In PRESET, it is quite certain that universities will keep the largest responsibility. In INSET, however, they are likely to lose a significant part of the responsibility they have traditionally had; this may be taken over by regional or local school communities.

4.2 In order to keep their leadership in PRESET and to improve the quality of preservice teacher education, the universities must first of all continue to "put their own house in order". It is clear, for example, that to make improvements in PRESET programmes with respect to A.3/A.4/A.5/A.6/A.7 (in part 1 of this presentation) is primarily our job as university people.

4.3 It is urgent that universities establish closer permanent connections and share some responsibilities with schools, both in order to improve the training of prospective teachers and to offer more relevant INSET courses and activities.

4.4 The really big challenge for universities in the future may well be to show enough imagination and initiative in offering new types of services and contributions to INSET (going far beyond offering credit-bound courses!), particularly if trends continue towards the establishment of "teachers' centres" and towards a greater integration of INSET activities with the professional task of the teachers. If they can achieve that successfully, I am convinced that universities will continue to play a key role in INSET although their responsibility will inevitably be somewhat diminished in this area.

Bibliography

Bélanger, Maurice. Innovation in in-service education and training of teachers: Canada. Paris: O.E.C.D., 1976. ERIC Microfiche ED 126045.

Commission on Preservice Education of Teachers of Mathematics of the National Council of Teachers of Mathematics. Guidelines for the preparation of teachers of mathematics. Reston, Virginia: N.C.T.M., 1973.

Daniel, John S. New models for in-service teacher training in Québec: The PERMAMA and PERMAFRA programmes. Paper presented at the Conference on Strategies for School-Focused Support Structures for Teachers in Change and Innovation, Stockholm, October 20-22, 1976 (sponsored by O.E.C.D., Paris).

Department of Education and Science. Teacher education and training. (A Report by a Committee of Inquiry appointed by the Secretary of State for Education and Science, under the Chairmanship of Lord James of Rusholme.) London: Her Majesty's Stationery Office, 1972.

Freudenthal, Hans. Teacher Training: An experimental philosophy. Paper presented at the Conference on the Problems of Teachers of Mathematics, Pécs, Hungary, August 23-27, 1977.

Gaulin, Claude. Some recent changes and trends in the philosophy and ways of organizing further training of secondary school mathematics teachers. Paper presented at the Conference on the Problems of Teachers of Mathematics, Pécs, Hungary, August 23-27, 1977.

Higginson, William. Training teachers of mathematics: Aspects of the Canadian experience. A "national survey" presented at the Conference on the Problems of Teachers of Mathematics, Pécs, Hungary, August 23-27, 1977.

Institut de l'UNESCO pour l'Education (Hambourg). Formation continue des enseignants de la mathématique au niveau secondaire. (Rapport d'une réunion internationale d'experts tenue à Hambourg en octobre 1968.) Paris: O.C.D.L., 1969.

IREM de Lyon. Bulletin inter IREM. Numéro spécial: PERMAMA. Lyon: Institute de Recherche dur l'Enseignement des Mathématiques, mars, 1977.

Otte, Michael *et al.* Tendencies and problems of the training of mathematics teachers. *Institut für Didaktik der Mathematik, Universität Bielefeld, 1975.*

UNESCO. New trends in mathematics teaching: Book IV (Chapter on trends in teacher training). Paris: UNESCO, 1977.

THE OBJECTIVES OF MATHEMATICS EDUCATION

A. J. Coleman

It would appear rather obvious that there could be only one objective of mathematics education. Clearly, it is to teach mathematics!

Agreed! But this having been said, many questions press upon one. What kind of mathematics? How much? To whom? Why teach any to anyone? Should the programme be the same for all pupils? If not, according to what criteria can pupils be distinguished? After all, in a democracy everyone is equal, so should we not aim to achieve a common universal mediocrity? Is there any percentage in considering objectives in a vacuum? Must we not link with such consideration an evaluation of the practical possibility of achieving them, given the mathematical competency (or incompetency) of the corps of teachers and the strong societal pressures which denigrate intellectual excellence or mental effort? In other words, is there any use discussing "objectives" without considering whether they can be achieved?

As with any issue of basic existential import, once you start thinking about the aims of mathematics education, questions flood into your mind. Clearly, I cannot in a brief article deal with them all or, indeed, adequately with even one of these many issues.

You will recall that fairly recently an OECD study of education in Canada expressed amazement at the extraordinarily high proportion of the GNP which goes into education in a society which seems to have no explicit statement of what it expects the school system to accomplish. Evidently most other OECD countries have clearer aims for education than has Canada. So perhaps my topic is timely.

Three objectives

I have read discussions in which as many as twelve aims of mathematics education were distinguished - or, at least, the author thought he could distinguish this number, though my mind was not sufficiently acute to grasp the subtleties of his thought. In any case, I consider such long lists pedantic nonsense and psychologically ineffective. It is best to concentrate on essentials. I fix on three aims which I regard as crucial.

1. The average citizen should be enabled to master the minimum of mathematics needed for ordinary commercial transactions and for understanding the functioning of our society.

2. All people should be given the possibility of enriching themselves culturally and intellectually by extending their basic human capacity for abstract thought.

3. A supply of thoroughly trained mathematical "practitioners" - engineers, economists, research mathematicians - sufficient for the needs of society should be assured.

Let us look at these objectives in order.

Of course, there cannot be a satisfactory dogmatic definition of the precise set of mathematical facts which every Canadian should know. A minimum would surely comprise arithmetic, including percentage; mensuration, including change of units; and simple geometry. However, anyone who has noted the extraordinary rate at which mathematics has increasingly been applied in our society in recent decades would argue that much more than this is needed. The proper functioning of democracy requires a universal ability to interpret, and not be misled by, a wide variety of statistics which various interest groups thrust upon us. Presumably the hand-held computer will be universally available for the generation of students which is now in the schools. They will need to learn how to use it effectively.

No matter how we define the contents of the curriculum required to accomplish the first objective, the material can properly be called basic. However, I am not prepared to campaign under the slogan "Back to Basics". We have never done an adequate job in inculcating basic mathematics in the average Canadian, and what might have been sufficient twenty years ago is not enough now and will be abysmally inadequate in another ten years. So I lift high my banner which reads: "Forward to the New Basics!"

In addition to enabling people to function as citizens or as economic agents in society, mathematics education should contribute to personal and cultural enrichment. David Wheeler has argued correctly that anyone who can use language effectively thereby demonstrates an ability to apprehend structure. But mathematics is the study of abstract structure, so one can argue that to mathematize is to be truly human. If, with Aristotle and St. Thomas Aquinas, we characterize man as the "rational animal", we must recognize that as we increase our ability to reason we enhance our humanness. Mathematical puzzles can be enjoyable and relaxing. But the more mathematics one understands and can use, the easier it is to understand and control the technological environment in which all of us are now immersed. Without such understanding the feeling that one's life is dominated by mysterious unknown forces must be overwhelming. So, to feel at ease with mathematics can enhance one's sense of freedom, as well as opening up the limitless and fascinating literature of mathematics and its applications.

All citizens should know enough mathematics to be able to manoeuvre in our society. All citizens should be given the opportunity to experience the joy of developing their innate capacity to mathematize and to exult in this power. But at a more mundane and practical level, Canadian society needs a supply of competent mathematical practitioners. Engineers, physicists, economists, biologists, and social scientists are increasingly making use of more and more sophisticated types of mathematics. They need to be properly trained. Many Canadians are proud that Bell Northern Research Company has a high reputation internationally as one of the few research-oriented companies in Canada which competes effectively for international markets. The Science Council of Canada Background Study, Mathematical Sciences in Canada, reports that of 126 professionals in BNR, two had doctorates, 26 had master's and 46 bachelor's degrees in mathematics. It is not a coincidence that the Canadian company with the highest research effectiveness is the one with the highest concentration of mathematically trained personnel.

If Canada wishes to remain in the forefront of the technological age, as it has in the past, it is desirable, indeed essential, that our programmes of mathematics education encourage gifted students to push their mathematical training forward as fast and as soundly as possible.

Are the objectives achievable?

I expect that most of my readers would agree that the three objectives sound fine.. However, doubts might be raised as to whether they are realizable, and questions will be asked about how they are to be achieved.

It is my belief that, at least in Western countries, our past methods of mathematics education have been abysmal. We have barely scratched the surface when it comes to developing and unleashing the power of human beings to mathematize. This is chiefly because for centuries the study of mathematics has been overshadowed by a powerful inhibiting factor which reveals itself in the wide-spread fear or awe of mathematics. Perhaps the root source of this fear is the manner in which the rote learning of Euclid was drilled into many successive generations of European, American, and Canadian children. Or perhaps its source is the stance of superiority which the mathematically-gifted, particularly university professors, have often assumed. Whatever the cause, I believe that a fear of mathematics and a feeling that "higher mathematics" (i.e. anything beyond 2 + 2 = 4!) is accessible only to a gifted elite have been the chief factors in preventing the majority of Canadians from entering with joy and satisfaction into the pleasant fields of mathesis.

This conviction was reinforced by my visit to the Soviet Union in April and May of 1977, when I had the opportunity to learn something about the "Kolmogorov reform" and to observe mathematics lessons in two schools at grade 3, 9, and 10 levels. Already in grade 3 (but not in 1 and 2), mathematics is taught by a specialist mathematics teacher. The system is not divided into elementary, junior, and senior schools. The child enters at the age of seven and continues in the same school for 10 years. All pupils study mathematics every year; six periods per week in grades 3 through 8, and five in grades 9 and 10. The textbooks and, in principle, the programme are uniform throughout the USSR. Apparently the reformed curriculum which has been gradually introduced during the past twelve years is fully implemented only in the cities. In School No. 169 in Leningrad, which specializes in English and not in mathematics, every pupil covers in ten years a mathematics syllabus which goes well beyond the total mathematics syllabus which is offered in thirteen years in Ontario to less than one-third of our children. I was particularly struck by the assigned homework on inequalities which was more difficult than we would dare set for a first year student at Queen's!

Are all children in Leningrad more intelligent than the top thirty percent of Ontario youth? I think not. In my view, the difference is that in Leningrad young people are better motivated and work harder. There is a basic confident feeling that everyone can and will learn mathematics. To be able to do mathematics is a natural human capacity which can be developed if one merely tries! The children are greatly helped by support and pressure from parents, reflecting the insistence of Lenin and his successors on the vital importance of the study of science in general, and of mathematics in particular, for the attainment of the social goals of the Communist Party.

There has been a great debate in North America about the so-called "New Math". In my view much of this has been ill-informed and misdirected. There is some evidence - but little which is statistically convincing - that students coming into the universities in recent years do not have as confident a control of the manipulative aspects of arithmetic and algebra as they did twenty years ago, This, it is claimed, proves that the New Math is a total disaster! I readily admit that some publishers who rushed on to the market with poorly written textbooks did a considerable disservice to mathematics education. Further, some teachers were ill-prepared to adopt the new approach to the teaching of mathematics which was introduced universally in the Soviet Union, Europe, and North America in the late 1950's. Some served up for their pupils a confusing mish-mash of poorly digested jargon.

However, many of the highly emotional critics of the "New Math" and the proponents of "Back to Basics" have merely added to the confusion without making any constructive contribution to the solution of the many important tasks of mathematics education in Canada.

They overlook key elements of our situation. The percentage of the school-leaving age group seeking entrance to university has increased markedly, so we are comparing the performance of the current freshmen with a smaller and more selected group. It has been estimated that nowadays the average child spends 15,000 hours watching TV. No one claims that this will improve mathematical competence! In Ontario, the abandonment of Grade XIII Examinations has had a greater influence - for good or ill - on the preparation of university freshmen than any other single factor with which I am familiar. I suspect that some of the older critics of the New Math tend to recall their youth through rose-coloured spectacles. From 1953 to 1960 I taught Freshman Calculus to the students of the Honour Course in Mathematics and Physics at the University of Toronto. They were the intellectual cream of Ontario - hardly excelled by a similar group anywhere in North America. In recent years I have taught analogous courses at Queen's. I cannot honestly say that during my 25 years experience I have observed any significant difference in the types of difficulties which students have had in understanding mathematics. I do recall a freshman at Toronto, in about 1955, who had come from the University of Toronto School with an average of 92 percent on the Grade XIII examinations, who thought that $(a + b)^{-1} = a^{-1} + b^{-1}$.

In Ontario, and I believe elsewhere in Western nations, there have been no essential changes in the mathematics curriculum in school between 1910 and 1960. However, in that period there was a total revolution in the role of mathematics in society. The aim of the old mathematics education was to inculcate the rote understanding of certain manipulative skills. In the 1950's we began to realize that this was not enough. In addition to basic manipulative skills the average citizen now needs to have some grasp, however dim, of what mathematics is, what you can expect of it and - equally important - what you cannot expect of it. Thus the aim of the new programmes is to convey an understanding of some mathematical ideas. Of course, this is much more difficult. The transition involves pain. It is far from complete. This is the contemporary challenge.

What can we do?

If our three objectives for mathematics education are accepted as necessary and desirable, then our first task is to make sure they are understood and accepted by the Ministers of Education, teachers, parents, and students. Only then can we hope to mobilize the forces needed to realize them effectively in Canada.

We must seek to dissipate the anxiety feelings towards mathematics, especially among elementary school teachers. This might be done by extensive in-service training programs and by improved pre-service courses. In order to ensure the latter, it is necessary for many university professors of mathematics to change their attitudes and redirect some of their energies. Possibly TV can be used to good effect - as has been done by PERMAMA in Québec and by Professor Z. Semadeni in Poland.

The crucial factor is that teachers should be competent and know and feel that they are competent. Then they will be psychologically free and able to open to their students the experience that to mathematize is to joy.

Working Group Reports

WORKING GROUP A: TEACHER EDUCATION PROGRAMMES

The mathematics educators and mathematicians in Working Group A pursued a variety of interesting and fruitful discussions. In retrospect, three major areas of concern emerged from these discussions:

1. Fully agreeing with C. Gaulin's position in his paper "Innovations in Teacher Education Programmes" (p.22), the group maintained that our universities are going to have to become much more involved in inservice programmes, using a variety of methods to meet teacher needs and developing a greater sensitivity to what these changing needs are. Some of the promising approaches to inservice training that are being experimented with in Canada at present are:

   a) Teachers centres. Universities in Québec, Ontario, and Manitoba are directly involved as initiators and/or continuing resource channels for teachers' centres in which local teachers are experiencing opportunities for "grass roots" curriculum development and professional growth that were inconceivable on the North American scene until a few years ago.

   b) Community-based, off-campus credit courses (university credit, salary credit, ...). A variety of methods for extending university services into surrounding and remote community centres are being experimented with. The PERMAMA project in Québec has developed an extensive set of inservice course modules on video-tape tailored to the expressed needs of over 4,000 secondary school mathematics teachers. Remote areas in Newfoundland are served by similarly videotaped inservice courses. Some of the modules use a multi-media approach. In several areas, master teachers from within given school districts have worked directly with university faculty members to develop and prepare themselves to teach inservice courses to their off-campus colleagues with and without videotaped modules. Such courses remain under the direct supervision of university faculty members and generate funds for the sponsoring institution.

   c) COLE boxes. The concept of these self-contained, self-instructional packages of multi-media resource materials, first designed for the "Creation of Learning Environments" by J. Trivett and made available to teachers by Simon Fraser University, is being used by C. Gaulin in Québec in the development of multi-media packages to provide teachers with resource ideas for classroom uses of electronic calculators.

d) Satellite campuses. Several universities have leased off-campus space to provide greater community access to university library facilities and "courses-on-demand".

e) Direct mathematics department contact with school teachers. The mathematics departments of the Université d'Ottawa and Carleton University, in collaboration with the Ontario Association for Mathematics Education, have offered inservice meetings for high school mathematics teachers, and Algonquin College has published a ten-issues-per-year problems journal entitled "Eureka".

There was general agreement that, in order to be effective in providing useful services to teachers, a heightened sensitivity to their needs at various levels is essential. For example, calculus courses are undoubtedly useful for those aspiring to teach high school calculus but there is a real need for more choices of suitable university courses in algebra, geometry, elementary mathematics from an advanced standpoint, number theory, history of mathematics, philosophy of mathematics, and problem solving.

2. There is a need for closer dialogue between everyone involved in the mathematics education community: for example, school board mathematics consultants, teachers, mathematicians, mathematics educators, and even the students themselves.

Outstanding classroom teachers should receive more respect and recognition and play a key role in the preparation of prospective mathematics teachers. J. Egsgard (see bibliography) has advocated extended internships for prospective teachers in which two or three work closely with a master teacher and his mathematics classes for an entire school year. Several universities are already seconding classroom teachers to help design and teach methodology courses (e.g., University of Calgary, Simon Fraser, Memorial). At the University of British Columbia, some professors have been able to switch roles with classroom teachers, each teaching one of the other's classes while continuing to draw their salaries from the usual sources.

Imaginative cross-fertilization of ideas among mathematicians, mathematics educators, and classroom practitioners would be encouraged by cooperative involvement in joint projects among resource personnel for teachers' centres and inservice seminars, workshops, and courses.

3. There is a real need for better communication across Canada with respect to innovations in teacher education practices in both pre-service and inservice programmes. For example, you may not know that:

The student teaching practicum at Simon Fraser is six months long during the professional year - four months in schools with regular seminars in the schools, two months on campus, and another two months in the schools. The university faculty members and staff associates (master teachers seconded to the university from the schools) conduct mathematics teaching seminars designed to give teachers learning experience in new contexts to exemplify creative uses of materials in teaching children. There is a deliberate attempt to blend theory and practice, mathematics and pedagogy.

At the University of Calgary, twelve classroom teachers in a variety of subject areas (two in mathematics) have been seconded to the university staff to assist in the teaching of methods courses, to provide continuous liaison with the schools and to conduct subject area and clinical supervision workshops. The mathematics teaching methods courses have a strong focus on active learning workshop experiences designed to generate enthusiasm for helping children learn mathematics using concrete materials. Similar emphases characterize the methods courses at the Universities of Alberta, Lethbridge, and Saskatchewan.

At the University of Manitoba, there is a choice of three pre-service professional year programmes for prospective teachers. The thirty-week professional year can be spent in a school-centred programme (22 to 23 weeks in schools), an integrated day programme (four half days per week in schools), or a faculty-centred programme (20 weeks on campus with a 10 week practicum). In the school-centred programme, student teachers spend one half day per week in teaching methods seminars.

The University of Toronto has initiated a teachers' centre experiment, as have Brandon University and the Université du Québec. At the University of Western Ontario, a semi-internship programme is under way in which prospective teachers spend two weeks in the last half of August in methods courses followed by full time involvement in the schools from September to December which, in turn, is followed by four months back at campus.

Laval is planning to initiate university-credit, school-based inservice courses to be taught by master classroom teachers. The elementary mathematics education students at Sherbrooke all take seven semester courses in *didactique mathématique*, three of which emphasize production of teaching units to be used in school classes during the practicum.

At Concordia University, the Université du Québec à Montréal, and the Université de Sherbrooke, mathematics education personnel are housed in the mathematics department. Concordia's offerings are primarily in the form of inservice programmes in which attempts are made to make every elementary mathematics education course an amalgam of mathematics and pedagogy. The impressive PERMAMA programme of the Université du Québec has already been mentioned.

In addition to the efforts to service remote areas, which have already been referred to, Memorial University offers a half course in teaching methods in which one hour per week is spent at the university and the equivalent of the other two hours per week is devoted to having each student teacher work with two to four school students in a mathematics learning activity designed for a specific classroom-related purpose. The school students are often selected because they are having remedial problems and they are taken out of regular classes for the individual or small group help.

A minor concern was expressed that in British Columbia, Alberta, and Saskatchewan, at least, a teaching certificate entitles one to teach any subject at any level. However, it was felt that, by and large, principals do assign teachers to teach subjects for which they are best qualified.

The point was also made that there is a need for university programmes for mathematics specialists in response to increasing needs for mathematics specialist coordinators for schools and school systems.

On the whole, the prospects for improved, flexible, service-oriented mathematics education programmes seem very good across Canada.

(Reporter: D.B. Harrison)

WORKING GROUP B: UNDERGRADUATE MATHEMATICS PROGRAMMES AND PROSPECTIVE TEACHERS

As one might have expected, in our group we disagreed as much as we agreed, and on every issue there were dissenters. We found it difficult to come to any specific recommendations or conclusions, and even the few we have to present (in my wording) were not all agreed to unanimously.

1. University faculty members should work more closely with teachers to learn their needs and assist them in their professional development. Examples of fruitful contacts were given.

2. It is desirable that work of this kind be recognized as a legitimate part of the activity of university faculty and rewarded appropriately.

3. We must find more ways of reaching teachers and providing a greater range of inservice education.

4. Any B.Ed. programme for elementary teachers at a university or college of education should include at least one mathematics class. The prerequisites for such a class should be determined by each province or university, and the course geared to the students' backgrounds.

5. Such a course should emphasize mathematical reasoning and problem-solving and should involve mathematical activity by the students as opposed to passive reception of sets of axioms, theorems, proofs, etc.

6. Each mathematics department should be encouraged to develop a suitable course and select instructors carefully.

7. A committee should be formed to develop some models of suitable courses for this purpose, including details of the topics to be covered and materials which would be useful.

8. We discussed at some length the question of whether "math majors", some of whom will likely teach at the secondary level, should take one of the existing programmes for majors, supplemented by a few special courses, or whether separate majors should be devised. We could not arrive at a consensus, partly because of the differences between provincial policies. Some of us felt that various options should be left open so that people of varying backgrounds could become teachers.

A number of topics were mentioned which, in addition to the usual calculus, geometry, linear algebra, etc., should be included in a major intended for teachers, e.g., statistics, mathematical modelling, combinatorics, history of mathematics, computing, etc.

9. Students should be exposed to a variety of types of reasoning and a variety of mathematics courses. Yet it is also important for students to get breadth as well as depth in some particular area of mathematics, not just a smattering of this-and-that.

10. The importance of the applications of mathematics was discussed. Teachers should have training in problem-solving and have available lots of good examples of non-trivial problems which they could use with their students. Such materials are slowly becoming available.

11. Some new, different, courses should be developed which concentrate on problem-solving, for example, in which the content is chosen according to the aim of the course - to encourage student participation, say. Another example of a different kind of course would be a semester course, at the end of the final year, reviewing and tying together all the mathematics previously learned.

12. University teaching should be improved.

(Reporter: C.H.M. Thomas)

WORKING GROUP C: RESEARCH IN MATHEMATICS EDUCATION

This group spent the three sessions considering two main questions:

1. What are the research concerns and dimensions of mathematics education research of the members of the working group?
2. What physical mechanisms can be set up for continuing communication among mathematics education researchers across Canada?

Even though the reward structures of faculties of education seems not to favour research activity, there is a large number of important projects under way, suggesting a certain vigour in the research efforts of the mathematics education community in this country.

Following is a sample of some of the research interests of members of the working group:

1. T.E. Kieren of the University of Alberta is making a study of rational number learning. This involves a detailed analysis of a personal rational number construct and of the development and constructive mechanism used in building up number ideas. One particular aspect of the research is obtaining evidence from careful interviews of children while they are involved in rational number tasks. Another is the attempt to assess the influence of a child's measurement notions on number learning and achievement.

2. J. Hillel of Concordia University is planning a study of the problem-solving behaviour of secondary school students using some of Krutetskii's problems in modified form. Part of the research involves an incorporation of some of Landa's heuristic advice.

3. J.E. Beamer of the University of Saskatchewan is making an assessment of the mathematics programmes in a school system and of the outcomes of these programmes. Baseline data on mathematical achievement were collected for children in the system. In a related study, the levels of basic mathematical competence of children in a province-wide sample were determined.

4. W.C. Higginson at Queen's University has under way an interdisciplinary project involving a psycholinguist, a computer scientist, two philosophers, and a mathematician, investigating cognitive and meta-cognitive aspects of children's mathematical knowledge. Children at four school levels, working in pairs, will be asked to work on some standard and some less orthodox problems. Analysis will be made of the language used by children and an attempt will be

made to categorize the children's mathematizing abilities. Influences include the work of Krutetskii, Landa, Ginsburg, and Skemp. In another project a new approach to teaching elementary number operations is being investigated, this research being influenced by the work of Wertheimer, Bruner, and Piaget.

5. D.F. Robitaille of the University of British Columbia has made an assessment of grade 4, 8, and 12 students' achievements in certain essential mathematical skills. Reports of the results are available from the Learning Assessment Branch, Ministry of Education, Victoria. A continuing project is concerned with the identification and correction of computational errors.

6. D.W. Alexander of the University of Toronto is investigating the nature of the difficulties in conceptualization which occur in children studying algebra. Analysis is based upon the imagery used and the fluency with which it is used.

7. To facilitate determination of student cognitive ability levels in particular mathematical contexts, D.B. Harrison of the University of Calgary has developed paper and pencil tests from clinical interview responses to ratio and proportion and mathematical reasoning problems. The instruments and strategies used in developing them should find application in helping teachers to match instruction to student ability and in evaluating and designing mathematics curricula.

8. L.D. Nelson of the University of Alberta has just completed a project concerned with determining the nature and development of problem solving behaviour in young children. This involved the video-taping and subsequent analysis of behaviour of children while they were engaged in solving non-verbal, highly concrete arithmetical problems. One feature of the children's behaviour was found to be their high susceptibility to distractions within the problem. This aspect is being studied systematically in a follow-up.

9. D. Lunkenbein of Université de Sherbrooke, using Piaget's concept of "grouping" (further formalized by Wittman), is studying how to rationalize teaching interventions. From this will be developed guidelines for the construction and analysis of actual teaching units.

10. F. Wan of the University of British Columbia brought to the attention of the group Dr. George Bluman's survey and analysis of data collected to clarify a number of issues associated with British Columbia secondary mathematics instruction and programmes, (e.g. non-comparability of school grades,

effect of sensitization, participation in provincial scholarships, examinations, etc.).

The lack of any effective communication links among the community of people interested in mathematics education research became apparent to the group, so consideration was given to how such links might be established and maintained.

(i) A newsletter will be established on a small scale to keep members informed of the research activity across Canada. This will be published periodically and will be coordinated initially by T.E. Kieren and D.F. Robitaille.

(ii) W.C. Higginson will investigate the possibility of using the Canadian Mathematical Congress' Notes to inform the mathematical community of research activities in mathematics education which might be of general interest.

(iii) D. Lunkenbein will investigate the possibility of arranging a meeting of researchers within the next year or two.

(iv) J. Hillel will determine the mechanism whereby scholars from the Commonwealth can be supported to visited Canadian universities. His research group is interested in visits by a number of British and French scholars doing research in mathematics education.

(v) Finally, more informal connections such as staff exchanges, summer school appointments, study leave visits among members of the group, will be facilitated where possible.

Behind all the activity, and the desire to extend it, however, remain two problems: adequate funding for mathematics education research, and recognition of its importance by university faculties of education.

(Reporter: L.D. Nelson)

WORKING GROUP D: LEARNING AND TEACHING MATHEMATICS

There was a considerable amount of general discussion whose chief value was to help the participants in this group gain some insight into their several approaches.

During the first session, we considered the learning process. Although some saw parallels between the mastery of mathematics and the mastery of music, athletics, or language, the lack of models and opportunities for pre-school explorations in the former made it difficult to motivate students and to establish standards of performance. It is important that the student of mathematics have a hand in defining his goals and in determining the extent of his involvement, that he has adequate time to gain experience and to develop intuition before being constrained by formalization and "correction" of his ideas into standard form, and that the authority to which he is subjected is that of the discipline rather than that of the teacher.

Our examination, in the second session, of the connection between the subject matter handled and the method of teaching it, dissolved into a discussion of the qualities of a good teacher. What seemed to bring to fruition the expertise, salesmanship, and personal characteristics of the teacher were the perspectives he can bring to his teaching and the model of a learner that he presents to his students. People at the university can foster the raising of philosophical, theoretical, and ethical questions. However, the effectiveness of the university in having an impact depends on the willingness of the faculty members to broaden their own horizons and improve their own teaching efforts.

To the question as to whether a teacher should be able to guarantee to his students that they will master certain material (at least if they put forth a suitable effort), the answer appeared to be a qualified yes, although there was much skepticism about performance-contract methods used in industry, and concern about the rise of competency-based teacher education in parts of the U.S.A. Would not a concentration on specific goals hamper deeper understanding of mathematics?

In the final session, some areas for possible research were listed:

(a) How can a teacher appreciate and exorcise the prejudices and misconceptions he may find in his students on a certain topic?

(b) Do we understand the nature of students' difficulties with notation? At what stage should we insist on standard notation? Fractions is a difficult area.

(c) How do we encourage the flexibility which allows a student to look at a mathematical idea from different standpoints as required? For example, parameters may be either variable or constant, or a point in a calculus or a locus problem may be either "stationary" or "moving".

(d) What strategies of learning can be taught? How can a body of material best be organized by the learner and assimilated?

(e) What is the best way to teach students how to solve multi-step mathematical problems?

(f) What can children with learning disabilities do in mathematics?

(g) How effective is "silent" teaching?

It was suggested that a list of mathematical stumbling blocks be compiled, that they be analyzed, and that techniques of dealing with them be recorded.

(Reporter: E.J. Barbeau)

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REFLECTIONS AFTER THE CONFERENCE

D. Wheeler

> *It no longer seems possible for any component of the mathematical ecosystem to function effectively in isolation. Awareness and communication seem to be the key issues.*
>
> Mathematical Sciences in Canada (p. 86)

They were the underlying themes of the conference too. Bringing university mathematicians and mathematics educators together involved an interaction between two groups which tend to be somewhat suspicious of each other. The assumption by the universities of the responsibility for training teachers has not led, in general, to greater mutual understanding or cooperation by those who teach university mathematics and those who teach would-be teachers of mathematics. Both groups have other interests and responsibilities and it may be that the lack of common ground in these other areas contributes to the suspicion. But it also extends into that part of their work where they might be expected to find a shared cause--the preparation of specialist mathematics teachers. University mathematicians look at education courses and see an apparent lack of structure and rigour together with a plenitude of non-refutable theories; university mathematics educators look at the students emerging from undergraduate mathematics programmes and see the apparently deadening effects of a training dominated by structure and rigour. Both sides, when apart, tend to stereotype each other. But when they get together the process of sorting fact from fantasy can begin. What each group is trying to do becomes clearer. It becomes apparent that the members of each group are as aware of some of the shortcomings in their own approach as they may be critical of the other's.

How does one give undergraduates an education in mathematics which suits the few who need a solid foundation for graduate work while catering for those who need some mathematics to apply in another field and those who need time to reflect on some of its fundamentals so that they can teach it better? How does an education programme integrate rather than juxtapose the many considerations that bear on the theory and practice of teaching? These are not easy questions for anyone to answer and it is not helpful merely to criticize without an appreciation of the difficulties.

Indeed, awareness of the complexity of the issues in even such a relatively small part of the mathematics education field as the preparation of specialist teachers is a necessary preliminary to any attempt at amelioration. Improvements may have to be tackled "in the small," but they do not therefore have to be simplistic. The advantage of bringing together two groups of people whose centres of interest do not quite coincide in order to discuss the same theme is that more of the complexity is likely to emerge and fewer simplistic statements are likely to survive.

But if two groups, why not three, or four, or ...? There are teachers, administrators, parents--and even students--who also have an interest in the preparation of teachers. Would not more complexity emerge, more awareness and communication take place?

"Perhaps, but maybe not," seems to be the only answer one can give to such a question. It would certainly dramatize another dimension or two of the complexity--that mathematics education is not only a number of programmes, or a set of ideas about the learning of mathematics, or a collection of skills and techniques for teaching it, but all of these, together with a host of organizations, institutions, and groups of people expressing various structures, investments, roles, values, expectations, and desires.

This is a situation that will defeat efforts to approach it analytically, attempts to distinguish all the elements and disentangle all the connections. We cannot solve problems in mathematics education by starting with an answer to the question, "What is the given?", because we can never know it in the usual, analytical sense. Such complexity must be embraced rather than analyzed, comprehended rather than understood. "Awareness" and "communication" are key issues, for sure, but what is it reasonable to expect of them? We can, and should, be aware of being plunged in complexity, though we will necessarily remain ignorant of most of its components; we live in a society with more means of communication than any other, yet mutual exchanges are invariably fogged with misunderstandings.

It is useful to remind ourselves at this point that we conduct the greater part of our daily lives under comparable conditions of ignorance and failures in communication, yet prove every day that they do not inevitably reduce us to impotence.

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None of this should be bad news for those of us who want to study and learn from the situation in mathematics education, or those who want to act on it in order to change it. Spelling out the conditions will help us avoid fooling ourselves or being fooled by others. They don't make us impotent. If we want to study children learning mathematics, for example, we fool ourselves by attempts to "control the variables," most of which we don't even know. All the elements of the complexity we are concerned with are potentially there in the responses of a few children--perhaps even of one child, who must himself be immersed in the complexities we have been talking about. We should not confuse complexity with size and scale. It may be extremely difficult to uncover "the universe in a grain of sand," but it is a better way of spending our time than by charting the deserts.

In considering how to bring about changes in mathematics education we have to recognize that we are dealing with a system with its own communication network, its own feedback mechanisms, and its own controls. If we act on it, something will happen, although we will not necessarily know in advance what that will be. (Consider the intentions and effects of the "new math" stimulus.) But if we understand that it has a life of its own, and is no longer--if it ever was--subject to commands, we will be alert to the movements that are taking place and act to reinforce those that we favour. Since we are ourselves "inside" the system, we will contribute to the gradual definition and realization of these trends. As members of the Conference heard, there are discernible movements that would give teachers a larger role in determining the form of their training, and would encourage them to participate in educational research. We will each decide, consciously or by default, whether to associate ourselves with these movements, and the choices we make in bodying forth our associations will help to determine the magnitude, direction, and form, of the development of the trends in the immediate future. For some, the observation that these trends signal a kind of interpenetration of roles (teachers becoming trainers and researchers, yet remaining teachers) will be ground enough for giving support, and will perhaps spur them to conceive other ways of blurring role differentiations, which may in turn become discernible movements ... and so on.

But this is already too simple, too rational-sounding, a description. We must continue to remember the more complex, opaque, nature of the situation.

Will these trends be short-term or long-term? Will they have large or small scale effects? It is almost impossible to be sure whether a particular perturbation is a temporary phenomenon or a more permanent shift. We have recently experienced the flow of "new math" and the ebb of "back to the basics," neither of which appears likely to leave much residue, to force anyone to look at mathematics education differently in the future. Are there underlying currents, though, which we could perceive if we were not too taken with the surface movements?

It is instructive to look for a moment at the changes in mathematics itself. If we try to account for the "explosion of mathematical knowledge which has taken place in the twentieth century"[1] we may see that a radical shift of awareness took place among mathematicians during the first half of the century. Mathematics, like any other study, is necessarily coloured by the beliefs that men hold about the world and their relationship to it. When men were mainly preoccupied with their relationship with the gods, or with God, mathematics was either magic, a mystery waiting to be interpreted, or it was a partial apprehension of the perfection of divine truth. When men turned their attention to the natural world, they saw mathematics as the expression of universal laws governing the terrestrial and cosmic worlds. But when they became increasingly conscious of their relationship with each other, mathematics adjusted itself again, emphasizing new notions of local validity and probable truth. This century has brought man right to the centre of things, with the realization that he creates the world he inhabits and that mathematics is one of the modes of mental activity he employs in constructing it. It is this new awareness which has liberated mathematics from physics and from metaphysics and led to an explosion of mathematical activity. Mathematicians now know themselves as people using their minds in a particular way, and since there are no restrictions on what they may think about, almost everything has become potentially mathematizable. Only the limitations of his imagination limit the mathematics man can make.

In one sense this development is a stage on a long road; in another it is clearly a new and different beginning.

Are there comparable changes to be detected in the field of education? The stages seem less clear, less ordered; yet we find clues if we study the various beliefs that have been held about the nature of man, for these have always had an inevitable impact on the theories and practices of education. What follows is a first, very rough, attempt to look in this direction.

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[1] Mathematical Sciences in Canada, p. 67.

In stable societies there have either been "caste" theories, implying different trainings for people destined by birth for different social roles, with "true" education reserved for an elite; or there have been more open, contrastingly egalitarian, theories stemming from religious beliefs which see children as equal at the starting-point but whose education serves to determine the adults they will grow up to be. In less stable societies, with the consequent awareness that neither social nor religious structures are absolute, comes the shift of attention to man as a learner and thinker, the idealistic image of rational man with its optimistic view of his potentialities. Education responds to this vision with secular schools, universities, and polytechnics, and rounds out the picture of the learned man, the scholar, whose work adds to the sum total of human knowledge and happiness. The optimism has to be countered, when it is seen that not everyone is able or willing to benefit from education, and pessimistic theories about the effects of "nature and nurture" on the individual are developed to explain the phenomenon. Education deliberately develops a differentiating function and becomes more and more allied to the production of a meritocracy.

The rapid expansion of mass education in the second half of this century, exposing the difficulties of providing an effective education for all in spite of the moral and political arguments in favour of it, and showing the inability of the meritocratic argument to keep its promises, has brought a critical, distrustful inspection of educational theories. In the midst of the confusion two responses seem to be developing. It is not surprising that one of these takes the form of an elaborate shrug expressing the impossibility of finding solid ground on which to build a system of educational theory and practice. It stresses the superficial aspects of individuality among children and makes a positive virtue of the uncertainty principle. Its contribution is that it puts the learner in the foreground and reminds us that it is the learner who has to do the most important job in the educational enterprise. Its assertion is that all children are different, which is a truism, and its practice is eclecticism, which is a cop-out. This view not only honestly confronts the fact that there is still a great deal at the heart of the teaching-learning process that is a mystery, but it also evades the possibility of penetrating it.

The second point of view now finding expression seems more positive and more hopeful. "The main aim [of education] should be that of exploiting, and extending, the ability to "mathematize" which is inherent in all thinking individuals."[2] Here the ability to mathematize is posited not as a differentiating ability that may or may not be part of one's genetic makeup, but as a general characteristic that comes with being human, like the ability to walk upright or to speak. This new awareness could liberate mathematics education by aligning it with the facts of mental activity and allowing it to be independent of many of the irrelevant preoccupations that have hampered it in the past.

This brief analysis is highly speculative and it would be premature to say that mathematics education is on the verge of a breakthrough comparable to that experienced by mathematics. Even if the signs have been read correctly, it will take time for the new awareness to irradiate the educational body, and we cannot predict how the system will respond. It took fifty years for more than a handful of mathematicians to notice what had happened to mathematics at the beginning of this century: the educational system has far greater investments in institutions, materials, bureaucracies, and beliefs.

Yet the real message of the implied parallelism is that there may be a current flowing that could liberate education from its ideological constraints. A hypothesis only, at the moment. But it may seem to some people a sufficiently rewarding prospect for them to want to accept it as a starting point. It is always a possibility that those who enter with curiosity and sensitivity and persistence into a dialogue with the facts may, like Kepler or Faraday or Cantor, find themselves carried into a new world that others will inherit.

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[2] Mathematical Sciences in Canada, p. 119.

Appendices

I Conference Participants

II Working Group Discussion Papers

III Conference Bibliography

IV Teacher Education in Mathematics:
A National Survey of Programmes

Appendix I

Conference Participants

D.W.M. Alexander
Faculty of Education
University of Toronto

H.A.J. Allen
Faculty of Education
Queen's University

E.J. Barbeau
Department of Mathematics
University of Toronto

J.E. Beamer
Faculty of Education
University of Saskatchewan

C.F.A. Beaumont
Faculty of Mathematics
University of Waterloo

M. Bélanger
Centre de la Recherche en Didactique
Université du Québec à Montréal

A.J. Coleman
Department of Mathematics and Statistics
Queen's University

J. Dhombres*
Conseiller Scientifique
Ambassade de France au Canada

J.C. Egsgard*
President, NCTM,
Twin Lakes S.S., Orillia

C. Gaulin
Faculté des Sciences de l'Education
Université Laval

D.B. Harrison
Faculty of Education
University of Calgary

W.C. Higginson
Faculty of Education
Queen's University

J. Hillel
Department of Mathematics
Concordia University

B.R. Hodgson
Département de Mathématiques
Université Laval

B. Johns
Faculty of Education
University of Manitoba

T.E. Kieren
Faculty of Education
University of Alberta

M.S. Klamkin
Department of Mathematics
University of Alberta

G. Lawson
Faculty of Education
University of Western Ontario

D. Lunkenbein
Département de Mathématiques
Université de Sherbrooke

I.E. MacNeill
Department of Mathematics
University of Western Ontario

S. McNicol
Faculty of Education
McGill University

L.D. Nelson
Faculty of Education
University of Alberta

F. Riggs
Faculty of Education
Memorial University of Newfoundland

D.F. Robitaille
Faculty of Education
University of British Columbia

J.M. Sherrill
Faculty of Education
University of British Columbia

P. Stewart
Department of Mathematics
Dalhousie University

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*Invited observer

P.D. Taylor
Department of Mathematics and Statistics
Queen's University

G.H.M. Thomas
Department of Mathematics
University of Saskatchewan

J.V. Trivett
Faculty of Education
Simon Fraser University

R. Vaillancourt
Département de Mathématiques
Université d'Ottawa

G. Vervoort
Faculty of Education
Lakehead University

F. Wan
Department of Mathematics
University of British Columbia

D.H. Wheeler
Department of Mathematics
Concordia University

Appendix II

Working Group Discussion Papers

(Written by the Organizing Committee and
distributed prior to the Conference)

WORKING GROUP A

Teacher education programmes

Preservice training programmes are not highly regarded by the teaching profession as a whole. Inservice programmes depend for their clients more on systems of incentives than on teachers' confidence in their value.

There is a general lack of evidence that might show whether teacher education programmes achieve the results they aim for; and, indeed, training departments are not always clear in saying what these aims are.

The present situation - economic restraint, high unemployment, a falling student population - inevitably urges a shift in the balance of training from preservice to inservice. Furthermore, the importance of inservice training, seen as "continuing education" is increasingly recognized.

"Updating"-style inservice programmes, boosted throughout the late sixties and the early seventies, particularly in mathematics, have in many cases only confirmed the effect of the content-oriented approach of the undergraduate and preservice programmes to which teachers had previously been exposed. Preservice B.Ed. programmes are as content-oriented on the education side as B.Sc. programmes on the mathematics side, the only difference being that the educational content is not so well-defined and not so subject to consensus.

There are two central problems that teacher education programmes must inevitably confront and which do not seem to be solved. One is the relation between the 'science' of education (pedagogy and, perhaps, more) and the 'science' of the subject matter that the students are teaching or will teach; the second is the reorientation of awareness that is involved in the (discontinuous) transformation of a learner into a teacher.

Suggestions for discussion

Current innovations and desirable developments in teacher education programmes.
The realization of teacher education as a form of 'continuing education'.
Associated questions: e.g. the relationship of universities with other interested parties, the place of apprenticeship in teacher education, the integration of mathematical and pedagogical components.

Chairman: R. Vaillancourt

WORKING GROUP B

Undergraduate mathematics programmes and prospective teachers

Undergraduate programmes have to serve a variety of purposes and a mixed group of students, only a few of whom will later become specialist teachers in schools. The problem of implementing these programmes and achieving reasonable justice for everyone, whether headed for teaching or for commerce and industry, or for more advanced mathematical study, is similar to the question facing the tailor: is it possible to make one size fit all? A solution may readily be found if one believes that there is a common core of mathematics that suits everybody; or, alternatively, that what matters is that students should learn how to learn and that they can be taught this through the medium of any mathematical content. Neither position is tenable in its pure form and one would expect to find a diversity of programmes, both within and between universities, reflecting different compromise positions. In practice there is more uniformity than one would expect. Undergraduate programmes seem, in fact, somewhat influenced by a collection of folk-criteria related to content, standards, comparability - and that mysterious quality, "credibility".

Students in some programmes in some institutions can graduate honourably without having encountered any history or philosophy of mathematics, having been exposed to any problems detached from the content of a particular course, having applied some mathematics to a non-mathematical situation, having tracked down references in the library, having discussed the social functions of mathematics, or having written a single word "about" mathematics.

Undergraduates are often left to do the most difficult learning job themselves - the job of tracing connections between topics belonging to courses with different labels. The instructors, whether specialists in the respective fields or not, do not generally see it as part of their responsibility to point out connections - and may have little time or incentive to do so. The forms of publicly evaluating students and instructors is a powerful factor influencing the way both parties spend their time with mathematics.

Without anyone intending it, the system of compulsory courses, elective assessment procedures and teaching styles often conveys a picture of mathematics that unduly stresses knowledge over insight and skills over know-how. The less capable students emerge with a view of mathematics which is not only partial but decidedly distorted. Attitudes may change and understanding grow when students move on into graduate programmes, but many of the future teachers will not go on to graduate school.

Suggestions for discussion

Current innovations and desirable developments in undergraduate programmes.
The balance between learning mathematics and learning about mathematics, or between learning mathematics and 'learning how to learn'.
Associated problems: e.g. making connections, the specific requirements of future teachers, improving the public image of mathematics.

Chairman: G.H.M. Thomas

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WORKING GROUP C

Research in mathematics education

Educational research, too, is held in low esteem. No doubt a good deal of it is trivial, but it is often judged against unreasonable claims and expectations. Big breakthroughs are not likely to be less rare than in other fields. Yet the suspicion remains that perhaps it hasn't yet had any at all and is still, in spite of its aping of scientific method, at the level of chemistry before Boyle or of astronomy before Kepler.

Thesis supervisors, journal editors and referees, tend to impose certain fixed forms on research projects and the communication of results. This often looks like a rule-game that a subset of mathematics educators plays, not to be confused with research in its full sense of noticing, describing, interpreting, and explaining what is the case concerning significant phenomena.

Educational research must be concerned that its findings be applicable, yet a case could be made that an excessive concern for applicability is partly responsible for the triviality (hence non-applicability) of a lot of it. If theories are reductionist, simplistic, jejune, as many of them are, the journeyman researcher has nothing to feed on. Searching, invigorating theories may alter awareness more effectively than the accumulation of quantities of results.

Where the core of educational research is concerned - learning, teaching, and the dialectic between them - the classroom is an adequate laboratory and there is nothing to stop the teacher doubling as researcher if he wants to.

The value of professional research for the teacher in the classroom may be that it alerts him to evidence he would not have noticed, indicates that there are options open to him that he might not have thought of, offers him conceptual schemas that he can use in making sense of his experience, and continually reminds him that there are always more questions to ask.

Suggestions for discussion

The present and foreseeable state of research in mathematics education.
What can reasonably be expected to emerge from research in this field.
Certain extrinsic problems: e.g. applicability, accessibility and 'criticizability'.

Chairman: L. D. Nelson

WORKING GROUP D

Learning and teaching mathematics

Teaching is neither a necessary nor sufficient condition for learning, yet institutionalized teaching behaves as if it were both. Students are *de facto* responsible for their own learning, yet they often feel incapable of accepting the responsibility and their teachers feel correspondingly reluctant to give it to them.

Language sustains the false picture. In North American English a teacher is an instructor, and there are no common equivalents of "didactique" or pedagogy. It would be more realistic to think of a teacher as a catalyst who may induce the precipitation of learning in his students, but we cannot even formulate this model in the current folk-language of education. Indeed, the catalyst-model is false in a different way as it doesn't suggest how a teacher must act or change himself to meet the variety of conditions in his students. The study of the function of teaching may have to remain at the allusive and metaphorical level until we have hammered out new concepts which will permit more adequate descriptions.

Everyone has the capacity to mathematize, but most do not know they have it. Not much is yet known about the mechanisms of mathematization, nor about the techniques by which it may be induced, but this nevertheless seems the most promising place to look to find the basis for a pedagogy of mathematics. Heuristic is a first step along the road, no doubt, but it seems better adapted to enticing students into active participation in learning than to giving them any precise information about their mathematical powers. And, even if it granted that mathematical powers exist in everyone, there is little sure knowledge about ways in which these powers can be engaged so that students learn the particular mathematics that they are given to learn.

The psychology of learning has not yet made much contribution to the study of teaching and is unlikely to do so until it abandons its fear of metaphysics and begins to articulate the functions of action, perception, imagery, language and symbolization in learning and makes bold inferences about the mental operations involved. The study of learning must be able to handle at least three varieties of data: observation of learning behaviours, introspection and personal experience, and rational analysis of the components present in successful learning of skills and knowledge.

Suggestions for discussion

The current and foreseeable state of the pedagogy of mathematics.
What we still need to know, and might be able to find out, about the learning of mathematics and the teaching of mathematics.
Related matters: e.g. innovations in teaching methods, a meta-language for talking about teaching and learning, mathematization as a power of the mind.

Chairman: E. J. Barbeau

Appendix III

Conference Bibliography

Participants were invited to recommend publications relevant to the conference themes. With some post-conference additions, the following materials constituted the Conference Library.

Books and Papers

Aitken, Lewis R., Jr. *Language Factors in Learning Mathematics*. Columbus: ERIC, Ohio State University, October 1972. 42pp.

Alexander, D. *The Case for Alternatives*. Toronto: Faculty of Education, University of Toronto, 1975. 8pp.

Alexander, D. *The JRME Policy*. Toronto: Faculty of Education, University of Toronto, 1975. 3pp.

Alexander, D.W. *Teacher Education Programs: Some Remarks*. Toronto: Faculty of Education, University of Toronto, 1977. 2pp.

Andrews, J.G. and McLone, R.R. *Mathematical Modelling*. London: Butterworth, 1976. 0408106018 260pp.

Apostol, Tom M. et al (Eds.). *Selected Papers on Calculus*. Washington: Mathematical Association of America, 1969. 397pp.

Apostol, Tom M. et al (Eds.). *Selected Papers on Precalculus Mathematics*. Washington: Mathematical Association of America, 1977.

Beamer, James E. *A Proposal for the Preparation of Teachers of Mathematics in Elementary (K-8) Schools*. Saskatoon: Faculty of Education, University of Saskatchewan. 5pp.

Beaumont, C.F.A. Observations on the Conference Programme. Excerpts from a letter. Waterloo: Department of Applied Mathematics, University of Waterloo, 1977. 2pp.

Begle, Edward G. (Ed.). *Mathematics Education*. The Sixty-ninth Yearbook of the National Society for the Study of Education. Part I. Chicago: University of Chicago Press, 1970. 467pp.

Begle, E.G. (Ed.). *The Role of Axiomatics and Problem Solving in Mathematics*. Washington: Conference Board of the Mathematical Sciences, 1966. 137pp.

Bélanger, Maurice. *Innovation in In-service Education and Training of Teachers: Canada*. Centre for Educational Research and Innovation (CERI). Paris: Organisation for Economic Co-operation and Development, 1976. 54pp.

Bell, A.W. *The Learning of General Mathematical Strategies*. Unpublished doctoral dissertation. University of Nottingham: Shell Centre for Mathematical Education, 1976. pb. 255pp.

Bell, A.W. and Wheeler, D.H. (Eds.). *Examinations and Assessment*. Mathematics Teaching Pamphlet No. 14. Nelson, Lancs: Association of Teachers ss of Mathematics, 1972 (1968). 110pp.

Beltzner, Klaus P., Coleman, A.J. and Edwards, Gordon D. *Mathematical Sciences in Canada*. Science Council of Canada Background Study No. 37. Ottawa: Science Council of Canada, 1976. 339pp.

Bluman, George W. A School-by-School Study Correlating Math 12 Grades and Performance in Math 100 for the 1976 Classes of Various B.C. High Schools. Vancouver: Department of Mathematics, University of British Columbia, 1977. 9pp.

Bluman, George W. Study of the Effect of Semesterization in Secondary Schools on Performance in First Year Calculus Courses at U.B.C. (1976 High School Class). Vancouver: Department of Mathematics, University of British Columbia, 1977. 7pp.

Bluman, George, W. and Smith, Warren C. Study of the Effectiveness of the 1976 Math 12 Scholarship Exams in Predicting Math 100 Results. Vancouver: Department of Mathematics, University of British Columbia, 1977. 6pp.

Burkhardt, Hugh (Ed.). University Mathematics Curricula and the Future. Proceedings of the Conference on "The Supply of Honours Mathematics Graduates", University of Nottingham: Shell Centre for Mathematical Education, 1977. 0906126002. 104pp.

Burkhardt, Hugh et al. Mathematical Education Review 1976-77. University of Nottingham: Shell Centre for Mathematical Education, 1977. 0906126010. 28pp.

Burnett, J.D. et al. Mathematical Knowledge: A Study of Cognition and Metacognition. A Research and Development Proposal. Kingston: Faculty of Education, Queen's University, 1977. 7pp.

Callahan, L.G. and Glennon, V.J. Elementary School Mathematics: A Guide to Current Research. Fourth Edition. Washington: Association for Supervision and Curriculum Development, 1975. 0871200767. pb. 188pp.

Cambridge Conference on the Correlation of Science and Mathematics in the Schools (1967). Goals for the Correlation of Elementary Science and Mathematics. Boston, Houghton Mifflin Co., 1969. pb. 208pp.

Cambridge Conference on Teacher Training (1966). Goals for Mathematical Education of Elementary School Teachers. Boston: Houghton Mifflin Co., 1967. pb. 138pp.

Canadian Mathematical Congress. Conference on Graduate Training of Mathematics Teachers. Papers from a Conference held at Sir George Williams University, Montreal, June 1969. London: William Clowes & Sons, 1972. 091955802X. Montreal: C.M.C., 1972. 110pp.

Cathcart, W.G. (Ed.). Mathematics Teaching: The State of the Art. Proceedings of the Edmonton Meeting of the NCTM, October 1973. Mathematics Council of the Alberta Teachers' Association (MCATA), Monograph No.2, September 1974. pb. 127pp.

Coleman, A.J. Mathematics in the Second Half of the Twentieth Century. Queen's Papers in Pure and Applied Mathematics, No. 27. Kingston: Queen's University, 1971. pp. 33-63.

Coleman, A.J. What is Mathematics? Is It Actually Difficult? Kingston: Department of Mathematics, Queen's University, 1977. 7pp.

Coleman, A.J. and Edwards, G. (Directors). Mathematics in Today's World. Proceedings of a Series of Seminars sponsored by The Mathematics Study. Ottawa: Science Council of Canada, 1974. pb. 360pp.

Committee on Support of Research in the Mathematical Sciences (COSRIMS) (Eds.). The Mathematical Sciences: A Collection of Essays. Cambridge, Mass.: M.I.T. Press, 1969. pb. 271pp.

Committee on the Undergraduate Program in Mathematics. A Basic Library List for Four Year Colleges. Second edition. Washington: Mathematical Association of America, 1977. 0883854236. pb. 106pp.

Conference Board of the Mathematical Sciences National Advisory Committee on Mathematical Education. Overview and Analysis of School Mathematics Grades K-12. Washington: Conference Board of the Mathematical Sciences, 1975. pb. 157pp.

Davis, R.B. Mathematics Teaching--With Special Reference to Epistemological Problems. Monograph No. 1, Journal of Research and Development in Education. Athens, Georgia: College of Education, University of Georgia, 1976. 71pp.

Egsgard, John. 'Mathematics Teacher Education: An Alternative'. NCTM Newsletter, December, 1976. p.2.

Fitzgerald, W.M. and Higgins, J.L. (Eds.). Mathematics Laboratories: Implementation, Research, and Evaluation. Columbus: ERIC, Ohio State University, November 1974. pb. 81pp.

Freudenthal, Hans (Ed.). Five Years IOWO. Special issue of Educational Studies in Mathematics, 7(3). Dordrecht: Reidel, 1976. pb. 367pp.

Freudenthal, Hans. Mathematics as an Educational Task. Dordrecht: Reidel, 1973. 9027702357. 680pp.

Gaffney, M. and Steen, L.A. Annotated Bibliography of Expository Writing in the Mathematical Sciences. Washington: Mathematical Association of America, 1976. 0883854228. pb. 282pp.

Gaskell, R.E. and Klamkin, M.S. The Industrial Mathematician Views his Profession: A Report on the Committee on Corporate Members. In American Mathematical Monthly, 81(7), 1974, 699-716.

Ginsburg, Herbert. Children's Arithmetic: The Learning Process. New York: Van Nostrand, 1977. 0442226950. pb. 197pp.

Glass, G.V. (Ed.). Science and Mathematics Education. Review of Educational Research, 39(4), 1969. Washington: American Educational Research Association.

Griffiths, H.B. and Howson, A.G. Mathematics: Society and Curricula. Cambridge University Press, 1974. 05109820. pb. 423pp.

Grothendieck, A. The Responsibility of the Scientist Today. (G. Edwards, trans.). Queen's Papers in Pure and Applied Mathematics, No. 27. Kingston: Queen's University, 1971. pp. 84-127.

Hadamard, Jacques. *The Psychology of Invention in the Mathematical Field*. New York: Dover, 1944. pb. 145pp.

Harrison, D.B. *Clinical Interviews on Piagetian Tasks*. Excerpts from the M.Sc. theses of B.A. Onslow and L.E. Tyrell. Calgary: Faculty of Education, University of Calgary, 1976.

Harrison, D.B., Onslow, B.A. and Tyrell, L.E. *The Real Dilemma in Secondary School Mathematics Education*. A paper delivered at MATHBEC 77, Montreal, May 1977. Calgary: Faculty of Education, University of Calgary. 19pp.

Higgins, Jon L. *Mathematics Teaching and Learning*. Worthington, Ohio: Charles A. Jones Publishing Co., 1973. 0839600135. pb. 228pp.

Higgins, Jon L. (Ed.). *Cognitive Psychology and the Mathematics Laboratory: Papers from a Symposium* (1973). Columbus: ERIC, Ohio State University. pb. 139pp.

Higginson, William C. *Reach and Grasp: Observations on the Potential and Reality of Mathematics Education*. Kingston: Faculty of Education, Queen's University, 1977. 17pp.

Higginson, William C. *Training Teachers of Mathematics: Aspects of the Canadian Experience*. Kingston: Faculty of Education, Queen's University, 1977. 15pp.

Hilton, Peter A. and Rising, Gerald R. *Thoughts on the State of Mathematics Education*. (33-42) in *Conference on Basic Mathematical Skills and Learning: Euclid, Ohio*. Volume two. Washington: National Institute of Education, 1975.

Hoechsmann, K. *On the Purpose and Results of the U.B.C. Basic Mathematics Test*. Vancouver: Department of Mathematics, University of British Columbia. 2pp.

Howson, A.G. *University Courses for Future Teachers*. (273-292) in *Educational Studies in Mathematics 6*, 1975.

Howson, A.G. (Ed.). *Developments in Mathematical Education*. Proceedings of the Second International Congress on Mathematical Education. Cambridge University Press, 1973. 0521098033. pb. 318pp.

Jackson, R.W.B. *Who Teaches What in the High Schools of Nova Scotia? Indicators of Imbalance in the Utilization of Talents and Training in the Teaching Force*. Unpublished paper. Halifax: Atlantic Institute of Education, July 1977. 57pp.

Kieren, Thomas E. *Overview and Analysis of School Mathematics Grades K-12*. (68-73) in *Journal for Research in Mathematics Education*. Review of National Advisory Committee on Mathematical Education. January, 1977.

Kieren, Thomas E. *Research on Computers in Mathematics Education*. The Use of Computers in Mathematics Education Resource Series. Columbus: ERIC, Ohio State University, April, 1973. 38pp.

Kilpatrick, J. and Wirzup, I. (Eds.). The Learning of Mathematical Concepts. Volume I of Soviet Studies in the Psychology of Learning and Teaching Mathematics. University of Chicago, 1969. pb. 216pp.

Klamkin, Murray S. On the Ideal Role of an Industrial Mathematician and Its Educational Implication. (244-269) in Educational Studies in Mathematics, 3. Dordrecht: Reidel, 1971.

Klamkin, Murray S. On the Teaching of Mathematics so as to be Useful. (126-160) in Educational Studies in Mathematics, 1. Dordrecht: Reidel, 1968.

Klamkin, M.S. and Newman, D.J. The Philosophy and Applications of Transform Theory. (10-16) in SIAM Review, 3(1), 1961.

Kline, Morris. Why Johnny Can't Add: The Failure of the New Math. New York: St. Martin's Press, 1973. 173pp.

Krutetskii, V.A. The Psychology of Mathematical Abilities in Schoolchildren. Chicago: University of Chicago Press, 1976.

Lakatos, Imre. Proofs and Refutations: The Logic of Mathematical Discovery, John Worral and Elie Zahar (eds.). Cambridge University Press, 1976. 0521290384. pb. 174pp.

Lancaster, Peter. Mathematics: Models of the Real World. Englewood Cliffs, N.J.: Prentice-Hall, 1976. 0135647088. 164pp.

Lesh, R.A. and Bradbard, D.A. (Eds.). Number and Measurements: Papers from a Research Workshop. Athens, Georgia: ERIC, Georgia Center for the Study of Learning and Teaching Mathematics, 1975. pb. 233pp.

Long, Richard S. et al. Research in Mathematics Education. (446-468) in Educational Studies in Mathematics, 2, 1970.

Lovell, Kenneth. Intellectual Growth and Understanding Mathematics. Columbus: ERIC, Ohio State University Information Center for Science and Mathematics Education, 1971. 21pp.

MacNeill, Ian B. Secondary School Statistics: A Survey. London, Ontario: Statistics and Actuarial Science Group, University of Western Ontario, 1977. 10pp.

Mason, J.H. and Baker, J.E. Mathematicking: The Distinction between Process and Content. Milton Keynes, U.K.: Department of Mathematics, Open University, 1976. 18pp.

May, Kenneth O. Teachers Should Know ABOUT Mathematics. (19-21). Conference on Graduate Training of Mathematics Teachers. Montreal: CMC, 1972.

McKenna, M.J. Scroggie, G.A. and Fox, D.A. 'Characteristics of Current Elementary School Mathematics Programs'. An Analysis of the Report with Recommendations. Toronto: Ministry of Education, 1976. 12pp.

Melzak, Z.A. Companion to Concrete Mathematics: Mathematical Techniques and Various Applications. New York: John Wiley & Sons, 1973. 0471593389. 270pp.

Mies, Thomas et al. Tendencies and Problems of the Training of Mathematics Teachers. Materials prepared by the working group 'Mathematiklehrerbildung'. Bielefeld: Institut für Didaktik der Mathematik, Universität Bielefeld, 1975. 135pp.

Ministry of Education and Ministry of Colleges and Universities, Ontario. Secondary/Post-Secondary Interface Study: Summary Report. Record No. ON00639. Toronto: Ontario Government Bookstore, 880 Bay St., 1977. pb. 136pp.

Morris, W. (Ed.). Effective College Teaching. American Council on Education, 1970.

NCTM. Guidelines for the Preparation of Teachers of Mathematics. Reston, Va.: NCTM, 1973. 32pp.

NCTM. A History of Mathematics Education in the United States and Canada. Thirty-second Yearbook. Washington: NCTM, 1970. 557pp.

NCTM Forum on Teacher Education, Chicago, April 1972. Promising Practices in Mathematics Teacher Education. Columbus: ERIC, Ohio State University, 1972. 210pp.

NIE (National Institute of Education). Conference on Basic Mathematical Skills and Learning. Volume I: Position Papers (227pp.) Volume II Working Group Reports (44pp.). Euclid, Ohio: NIE, 1975. pb.

OAME. A Brief on the Qualifications and Preparation of Teachers of Mathematics, Grades K-8. A position paper sent to the Minister of Education, June 1974. 25pp.

OECD (Organisation for Economic Co-operation and Development). Reviews of National Policies for Education: Canada. Paris, 1976. 9264115455. pb. 264pp.

OISE. Geometry: Kindergarten to Grade Thirteen. Toronto: OISE, 1969 (1967). 119pp.

OISE. K-13 Mathematics: Some Non-Geometric Aspects. Part I: Statistics and Probability. Toronto: OISE, 1971. 077440048X. 56pp.

OISE. K-13 Mathematics: Some Non-Geometric Aspects. Part II: Computing, Logic and Problem-Solving. Toronto: OISE, 1971. 077440048X. 61pp.

Ontario Ministry of Education. The Formative Years. Circular P1J1. Provincial Curriculum Policy for the Primary and Junior Divisions of the Public and Separate Schools of Ontario. Toronto: OME, 1975. 24pp.

Ontario Ministry of Education. Education in the Primary and Junior Divisions. Toronto: OME, 1975. 110pp.

Osborne, Alan (Ed.). An In-Service Handbook for Mathematics Education. Reston, Va.: NCTM, 1977. 0873531191. pb. 259pp.

Osborne, A. and Suydam, M. Investigations in Mathematics Education, 10(3). Columbus: The Center for Science and Mathematics Education, ERIC, Ohio State University, 1977.

Ost, David H. (Coordinator). Guidelines and Standards for the Education of Secondary School Teachers of Science and Mathematics. AAAS Miscellaneous Publication 71-9. Washington: American Association for the Advancement of Science. 77pp.

Otte, Michael, Reiss, Veronika, and Steinbring, Heinz. The Education and Professional Life of Mathematics Teachers. Bielefeld, Germany: Universität Bielefeld, 1975. 44pp.

Polya, George. Mathematical Discovery: On Understanding, Learning, and Teaching Problem-Solving. New York: John Wiley & Sons. Volume I 1967 (1962) 216pp. Volume II 1968 (1965) 213pp.

Rensselaer Polytechnic Institute Symposium on 'The Mathematical Training of the Nonacademic Mathematician', 1974. Education in Applied Mathematics. (541-558) in SIAM Review 17(3), 1975.

Robitaille, D.F. and Sherrill, J.M. et al. British Columbia Mathematics Assessment, 1977, Summary Report. Victoria: Department of Education, British Columbia. 59pp.

Romberg, Thomas (Chairman). Report on the Working Group on Research Priorities. (22-32) in Conference on Basic Mathematical Skills and Learning, Euclid, Ohio. Volume Two. Washington: National Institute of Education, 1975.

Rosskopf, M.F., Steffe, L.P. and Taback, S. Piagetian Cognitive-Development Research and Mathematical Education. Proceedings of a conference at Columbia University, 1970. Washington: NCTM, 1971. pb. 241pp.

Russell, H.H., Robinson, F.G., Wolfe, C. and Dimond C. Current Ontario Elementary School Mathematics Programs. Record No. ON00193. Toronto: OISE, 1975. pb. 305pp.

Simon, H.A. Learning with Understanding. Mathematics Education Information Report. Columbus: ERIC, Ohio State University, 1975. pb. 25pp.

Skemp, Richard R. The Psychology of Learning Mathematics. Harmondsworth: Penguin Books Ltd., 1971. 0140213104. pb. 319pp.

Suydam, Marilyn N. A Categorized Listing of Research on Mathematics Education (K-12) 1964-1973. Columbus: ERIC, Ohio State University, August 1974. pb. 358pp.

Suydam, Marilyn N. Compilation of Research on College Mathematics Education. Columbus: ERIC, Ohio State University, December 1975. pb. 285pp.

Suydam, Marilyn N. Evaluation in the Mathematics Classroom: From What and Why to How and Where. Columbus: ERIC, Ohio State University, January, 1974. pb. 62pp.

Suydam, Marilyn N. Teaching Mathematics to Disadvantaged Pupils: A Summary of Research. Columbus: ERIC, Ohio State University, April, 1971. pb. 65pp.

Suydam, M. and Dessart, D. Classroom Ideas from Research on Computational Skills. Reston, Va.: NCTM, 1976. pb. 58pp.

Taylor, Peter D. A Red Book for Queen's. Kingston: Department of Mathematics, Queen's University, 1976. 51pp.

Taylor, Peter D. The Structure of Text Books. Kingston: Department of Mathematics, Queen's University, 1977. 2pp.

Travers, K.J., Pikaart, L., Suydam, M. and Runion, G.E. Mathematics Teaching. New York: Harper & Row, 1977. 0060452331. 591pp.

Trivett, John V. Forward to the Basics. (4-11) in Mathematics Teaching 79, June 1977.

Trivett, John V. Towards a New Design for Learning How to Teach Mathematics: Some Comments Regarding Innovation in Teacher Education. Burnaby: Simon Fraser University, August 1977. 3pp.

Trivett, John V. Which Researchers Help Teachers Do Their Job? (39-43) in Mathematics Teaching 78, March 1977.

Watson, F.R. Developments in Mathematics Teaching. London: Open Books, 1976. 0729100804. pb. 142pp.

Weygang, P. Mathematics Curricula in Canadian Senior High Schools: A Global and Provincial Comparative Study. Unpublished paper. Kingston: Faculty of Education, Queen's University, 1975. 94pp.

Wheeler, David. Humanizing Mathematical Education. (4-9) in Mathematics Teaching 71, June 1975.

Wheeler, David. Mathematization. (20-23) in Recognitions 7, October, 1976. Reprinted from Notes of the Canadian Mathematical Congress, October, 1974.

Wittenberg, A. An Unusual Course for Teachers of Mathematics. (1091-1097) in American Mathematical Monthly, December 1963.

Wittrock, M.C. Recent Research in Cognition Applied to Mathematics Learning. Columbus: ERIC, Ohio State University, February 1973.

Journals

Bajpai, A.C. and Martin, W.T. (Eds.) International Journal of Mathematical Education in Science and Technology 3(3), 1972. Chichester: Sussex: Wiley-Interscience.

Davis, R.B. and Ginsburg, H. The Journal of Children's Mathematical Behavior. The Madison Project, Supplement No. 1. Champaign, Ill.: Study Group for Mathematical Behavior, Inc., 1976. 324pp.

NCTM. Journal for Research in Mathematics Education 9(1), January 1978. Reston, Va.: NCTM.

OAME. Ontario Mathematics Gazette, 15(3), March 1977.

Saskatchewan Mathematics Teachers' Society. News/Journal 15(3), Summer 1977.

Appendix IV

Teacher Education in Mathematics: A National Survey of Programmes

It was the decision of the Conference that it would be worth compiling, in summary form, outlines of Teacher Education Programmes in each of the Canadian provinces. Accordingly, information has been coordinated and submitted by conference participants: Sherrill (British Columbia), Harrison (Alberta), Beamer (Saskatechewan), Alexander (Ontario), Wheeler (Québec), Stewart (New Brunswick, Nova Scotia, Prince Edward Island), and Riggs (Newfoundland). Information for the province of Manitoba was not available.

Educating Teachers of Mathematics
in the Province of British Columbia

Summary

	Elementary	Secondary
Preservice	Degree: 4 year B.Ed., or one year certification after the B.A. or the Diploma programme. Courses: Education: 10 2-semester Math Ed: SFU - none; UBC - one 2-semester; UVic - one 1-semester Mathematics: SFU, UBC - none; UVic - 2 semesters. Practica: SFU - 18 weeks divided 6 and 12; UBC - 10 weeks divided 3, 4, 3; UVic - 11 weeks divided 2, 3, 6. # of students: SFU - 820; UBC - 1325; UVic - 320. # of schools: SFU - data not readily available; UBC - 135 (fall) and 700 (spring); UVic - 75(fall) and 125 (spring). # of teacher slots for year: SFU - data not readily available; UBC - 6000; UVic - 600.	Degree: 5 year B.Ed. or one year certification after B.A. or the Diploma programme. Courses: Education: 8 2-semester Math Ed: SFU - none required; UBC - 2 semesters required (but 3 semesters taken) UVic - 4 semesters required. Mathematics: SFU - none required but they should have at least a minor. UBC - 7 semesters; UVic - 8 semesters. Practica: Same as elementary. # of students: SFU - 30; UBC - 20; UVic - 20. # of schools: SFU - data not available; UBC - 10 (fall), 20(spring) UVic - 8 (fall), 15 (spring). # of teacher slots for year: SFU - data not available; UBC - 45; UVic - 25.
Inservice	All three universities have very small graduate programmes. UBC offers the M.A., M.Ed., and Ed.D., but has only about 5 graduate students on campus and about 10 off campus. All three universities take their inservice work off campus into the interior of British Columbia. SFU, as a university, has been very active in off campus courses. The courses offered by UBC's and UVic's mathematics education groups have been very well attended, but the number of courses is very limited.	All three universities have very small graduate programmes. The inservice work takes the same form as for the elementary teachers. The number of secondary teachers contacted, however, is much smaller than the number of elementary teachers.

SFU = Simon Fraser University
UBC = University of British Columbia
UVic = University of Victoria

Educating Teachers of Mathematics in the Province of British Columbia

General Comments

There are only three universities in the province and all three train mathematics teachers. The programme at UVic and UBC have many similarities while the programme at SFU is organized very differently from the other two universities. The SFU programme includes two practica totalling 18 weeks while the UBC and UVic programmes divide 10 and 11 weeks of student teaching into three practica. The longest practicum at UBC is 4 weeks, at UVic is 6 weeks, and at SFU is 12 weeks.

UVic has a secondary internship programme designed to integrate theory and practice and to increase the amount of time the students spend in the schools. UBC has nine alternate programmes which restructure the regular programme. One characteristic of the alternate programme is a drastic increase in the length of the student teaching experience. Some of the alternate programmes also integrate the mathematics content and methods for the elementary education majors.

The "in term" practica of the three universities put a real strain on the selection process for cooperating teachers. Many teachers have to be used almost continuously to supervise student teachers.

At entry to the Professional Development Programme at SFU it is estimated that less than one-fifth of all elementary school-bound trainees have studied mathematics content since their own secondary school experience. About one-half elect to take some mathematics seminars/workshops during the year's training, spending between ten and thirty-six hours on such work. Such variations depend on other duties of available and qualified instructors and the plethora of alternative student choices to fulfill individual programmes.

Controlling variables include the difficulties of geographical dispersion of external-to-campus satellite programmes, only three faculty members to deal with mathematics and priorities given to alternative subject matter and methods, particularly in alleged response to current cultural wishes, namely learning disabilities, reading, ESL, core curriculum, behavioural objectives, classroom process analysis, etc.

It is the existence of the Professional Development Programme as opposed to a professional year within a degree programme that accounts for the appearance of no mathematics being required of the elementary education majors at SFU.

Educating Teachers of Mathematics
in the Province of Alberta

Summary

	Elementary	Secondary
Preservice	Semesters of study for Education degree: 8 Number of Education semester courses required: 16 (U of L) to 23 (U of C) Minimum number of Mathematics semester courses required: 0 Minimum number of Mathematics Education semester courses required: 1	Semesters of study for Education degree: 8 Number of Education semester courses required: 16 (U of L) to 18 (U of C) Minimum number of Mathematics semester courses required: 8 (U of L) to 10 (U of C) Minimum number of Mathematics Education semester courses required: 2
Inservice	Mathematics Council of the ATA Regional Offices at the Provincial Department of Education Profession-sponsored inservice days School district-sponsored inservice University: Summer school and evening courses in Mathematics Education	

U of L = University of Lethbridge
U of C = University of Calgary

Educating Teachers of Mathematics in the Province of Alberta

General Comments

Teacher Education Institutions: University of Alberta (U. of A.)
University of Calgary (U. of C.)
University of Lethbridge (U. of L.)

U. of A. and U. of C. offer masters' and doctoral level programmes in elementary and secondary mathematics education.

At all three institutions, the student-teaching practicum consists of ten week of full days in the schools plus a preliminary one-day-per-week observation round for six weeks.

U. of C. has thirteen classroom teachers (two in mathematics) seconded as "university associates" (U. of A. has seven) to assist in the teaching of methods courses and in maintaining university schools' liaison with respect to the student teaching practicum.

At U. of L. teaching methods courses in mathematics are common for both elementary and secondary prospective teachers. Assignments vary according to student interest and there is specific instructional follow-up during the practicum.

Educating Teachers of Mathematics in the Province of Saskatchewan

Summary

	Elementary	Secondary
Preservice	Semesters required for B.Ed.: Saskatoon: 8 Regina: 8	Semesters required for B.Ed.: Saskatoon: 8 Regina: 8
	Minimum semesters required for certification: Saskatoon: 6 Regina: 6	Minimum semesters required for certification: Saskatoon: 8 Regina: 8
	Minimum number of semester Education courses for certification:* Saskatoon: 16 semesters + internship Regina: 14 semesters + internship	Minimum number of semester Education courses for certification:* Saskatoon: 12 semesters + internship Regina: 6 semesters + internship
	Minimum number of semester university Mathematics courses for certification:* Saskatoon: 2 Regina: 1	Minimum number of semester university Mathematics courses for certification:* Saskatoon: 10 Regina: 8
	Minimum number of semester Mathematics Education courses for certification:* Saskatoon and Regina: 1	Minimum number of semester Mathematics Education courses for certification:* 0 for non minors Saskatoon and Regina: 1

Inservice

Inservice is mainly the function of the school district. It usually takes the form of 2 institute days a year, several days devoted to building level activities, after school inservice meetings, and committee work. These institute days must allow for inservice in all subject areas.

In rural areas inservice is a function of the school unit. The planning of the allowed institute days is by the district superintendent and the Professional Development Committee as dictated by the needs of the Unit. Since there is frequently only one mathematics teacher in a school, building level activities are minimal. In some jurisdictions the mathematics teachers of the unit meet at regular intervals.

Several other organizations are involved in mathematics inservice; examples are the Saskatchewan Teachers Federation and the Saskatchewan Mathematics Teachers Society. The Department of Education has done minimal work on programme development and implementation of new programmes. Their contribution to metric education has been more adequate because of their commitment to a full time metric-education consultant and a more extensive programme of workshops.

For those students enrolling in advanced undergraduate or graduate programmes, the universities play some role in keeping teachers up-to-date. Also university faculty are active in local and provincial committees, and in the Mathematics Teacher's Society. Both universities have graduate mathematics education programs.

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*These courses are semester courses, roughly equivalent to one-half class or 3-4 semester hours in length.

Educating Teachers of Mathematics in the Province of Saskatchewan

General Comments

Many secondary majors take two undergraduate Math. Ed. courses. Many minors take at least one. However, because of blanket certification policies, a teacher can get a two or three year certificate for elementary or secondary and then teach at the other level with no special training for these duties. In fact, once certificated, a teacher can teach anything to anybody at any level, except possibly for a few grade 12 academic classes. Hence there is no way of ensuring teachers assigned have the special background that is desired for teaching the subject or level. However, the Department of Education points out that in both elementary and high schools:

- there is some onus upon the employer to place teachers in positions for which they are best trained.
- there may be an over-emphasis on training teachers for specific subject areas. Perhaps the emphasis should be on how to teach children rather than on how to teach a subject. Obviously a combination of the two is necessary, but one aspect should not be sacrificed at the expense of the other.
- since many of our schools in Saskatchewan are relatively small, we do require teachers who are generalists or who have competencies in several subject areas. This raises the question of how much training in a specific subject area a teacher really requires in order to adequately teach that subject.

Certification procedures for elementary school teachers have been revised recently. Both campuses are busy revising programmes and experimenting with alternatives. The provincial Department of Education currently has one full time math-science consultant (K-12) and one full time metric consultant.

Two Universities in Saskatchewan prepare teachers. The University of Saskatchewan recommends about 375 teachers for certificates each year. The University of Regina recommends about 25 teachers per year. The split is about 60% elementary, 40% secondary. Currently about 220,000 students are enrolled in grades K-12 in Saskatchewan.

Educating Teachers of Mathematics in the Province of Ontario

Summary

	Elementary	Secondary
Preservice	2 semesters after B.A. or B.Sc., (except for some small programmes in which B.Ed. is taken concurrently with B.A.). 8 education courses (17-20 weeks). No university math courses required as prerequisite. 18-34 hours of mathematics education. Ministry of Education grants certificate. 8 week practicum is minimum requirement.	2 semesters after B.A. or B.Sc. (except for some small programmes in which B.Ed. is taken concurrently with B.A.). 8 education courses (17-20 weeks). 2 university math courses required as prerequisite. 51-119 hours of mathematics education. Ministry of Education grants certificate. 8 week practicum is minimum requirement.
Inservice	Various certificate programmes available. Elementary teacher may qualify as Mathematics Specialist by taking 6-9 university mathematics courses plus a seminar in mathematics education. Summer programme provides secondary teacher with opportunity of becoming certified at elementary level. (This is a 'Teacher Centre' type of programme.) M.Ed. programme offered by O.I.S.E., University of Western Ontario.	Secondary Teacher may qualify as Mathematics Specialist by taking 6-9 university mathematics courses plus a seminar in mathematics education. There is a summer programme for elementary teachers to become certified as secondary mathematics teachers. M.Ed. programme offered by O.I.S.E. M.A.T., M.Ed. programmes offered by the University of Western Ontario. The University of Waterloo provides correspondence courses which may be used by teachers to qualify as Mathematics Specialists.

Educating Teachers of Mathematics in the Province of Ontario

General Comments

There are eleven institutions involved in preservice teacher education in Ontario and the programmes offered include programmes involving internships, those providing no mathematics education courses, those permitting candidates to qualify as elementary and secondary teachers simultaneously, and those essentially based on school experience rather than university course. All programmes leading to certification at the secondary school level demand qualification in two teaching subjects although the certificate permits the holder to teach any subject. All programmes leading to certification at the elementary school level include experiences in all subjects taught at that level.

Teacher education institutes in Ontario:

Faculties of Education: Lakehead, Laurentian, Brock, Windsor, Western Ontario, Toronto, York, Queen's, Ottawa.

Ontario Teacher Education Colleges: Hamilton, Toronto.

Ontario Institute for Studies in Education.

Educating Teachers of Mathematics
in the Province of Québec

Summary

	Elementary	Secondary
Preservice	6-semester bachelor programmes variously titled, or 2-semester post-bachelor Diploma. The great majority of students qualify under the first alternative. In general, about one-tenth of the bachelor programme consists of compulsory mathematics and/or mathematics education courses.	6-semester bachelor programmes, variously titled, or 2-semester post-bachelor Diploma. A majority of francophone students qualify under the first alternative; a majority of anglophones under the second. Up to two-thirds of the bachelor programme may consist of mathematics and/or mathematics education courses, the remainder being education.
Inservice	Certified teachers may enrol in the preservice bachelor programmes Certificate programmes are available at some universities. Concordia and Sherbrooke Universities offer specially designed master's programmes for elementary teachers of mathematics.	Certified teachers may enrol in the preservice bachelor programmes. Certificate and Diploma programmes are available at some universities. Concordia, Laval, and Montréal Universities offer master's programmes in the teaching of mathematics. PERMAMA

Educating Teachers of Mathematics in the Province of Québec

General Comments

Thirteen institutions (including the four campuses and the Téléuniversité of the University of Québec) offer preservice and/or inservice programmes for teachers of mathematics. In some of these (for example, the University of Québec in Montreal, Sherbrooke and Concordia Universities) the Department of Mathematics rather than the Department or Faculty of Education is responsible for some or all of the pedagogical components in these programmes.

The majority of inservice programmes at any level can be followed on a full-time or a part-time basis. Certain preservice programmes are also available to part-time students.

Teaching certificates are awarded by the Ministry of Education; they are generally specific to a particular level of teaching and, at the secondary level, to a particular subject or group of subjects.

In the Province of Quebéc, students attend a two-year college (CEGEP) after completing Grade 11 in secondary school and before entering university. Bachelor programmes normally require three years (six semesters, 90 credits) of study.

The table refers only to university involvement in inservice training. Inservice activities are also organized by the Ministry, school boards and professional associations. Each school board allocates a number of pedagogical days (between six and ten each year) which are normally used for school-board inservice activities.

Educating Teachers of Mathematics in the Province of New Brunswick

Summary

Université de Moncton
(francophone system)

<table>
<tr><th></th><th>Elementary</th><th>Secondary</th></tr>
<tr><td rowspan="2">Preservice</td><td>8 semesters are required for a degree in education.
Number of education courses: 27 one-semester courses, including 1 one-semester course of math education and 4 one-semester courses of math.</td><td>8 semesters are required for a degree in education.
11 one-semester courses of education.
1 one-semester course of math education.
11 one-semester courses of math for a degree with concentration in math.</td></tr>
<tr><td colspan="2">The education courses and mathematics education courses are given by the Faculty of Education (Université de Moncton).
The mathematics courses are given by the Department of Physics and Mathematics (Université de Moncton).</td></tr>
<tr><td>Inservice</td><td>There are no courses in mathematics especially given for elementary teachers.</td><td>There are no courses in mathematics especially given for the secondary teachers. Generally they take courses in college algebra, elementary calculus, elementary statistics, etc. These courses are given by the Department of Physics and Mathematics (Université de Moncton).</td></tr>
</table>

General Comments

The university has no consultation service as such for mathematics teachers. However any requests for conferences, workshops, etc., received from secondary schools, are usually met by the physics-math department.

At the graduate level the masters in education depend entirely on the education faculty.

The training of elementary school teachers may be regarded as adequate since there is no specialization required at this level. On the secondary level, the new programme contains eleven one-semester courses in mathematics (for those who chose a bachelor of education with a major in mathematics). However, very few students take this programme. In New Brunswick, l'Université de Moncton is the only institution that has the responsibility to grant certificates to francophone teachers.

Educating Teachers of Mathematics in the Province of New Brunswick

Summary University of New Brunswick

	Elementary	Secondary
Preservice	Each student must complete 12 credit hours of mathematics education including: 3 hours of content orientation, 3 hours of an overview of the mathematics underlying the elementary school curriculum, 3 hours of instruction in the specific methods and materials for either primary or intermediate grades, and 3 hours of general math methodology. A student concentrating in math completes an additional 12 credit hours in math and math education, at least 6 in mathematics itself.	A major must complete 6 full mathematics courses (at least 24 credit hours at upper levels) as well as a 6 credit hour methodology course. A minor must complete at least 18 credit hours of mathematics as well as normally completing 3 credit hours of methodology. There is an additional option of a a methods course specifically in junior high mathematics.
Inservice	No formal arrangements exist between the university and the school system. However, it is recognized by the faculty of the university that there is an obligation to provide inservice as requested by school districts where practicable. A member of the university faculty sits on each provincial math curriculum subcommittee and is, therefore, involved in provincial inservice as well. Summer and extension courses as well as graduate programmes are normally provided in mathematics to act as one way of updating mathematics teachers.	

General Comments

There are two main institutions responsible for teacher training in New Brunswick--the University of New Brunswick for English speaking teachers, and the University of Moncton for French speaking teachers. There is also a special programme at St. Thomas University and one at Mount Allison University, but they are small in comparison. The data included in the survey reflects the policy at the University of New Brunswick only.

There are two main routes for gaining a teaching licence at UNB: the B.Ed. as a first degree and the B.Ed. as a second degree. Although the order in which courses are done will vary with the two choices, each student is normally required to complete all courses listed on the other sheet. There are exceptions made and individualized programmes are sometimes worked out. The variation from the courses listed here is usually minimal.

A special concentration in junior high is offered at the secondary level. A student chooses two main subject areas (one of which may be mathematics) and takes special courses in junior high methodology. Here only five courses may be demanded in the 'major' area, rather than six.

There are approximately 107,682 English speaking public school students in the province in approximately 502 schools (French and English). The total number of anglophone teachers is 5029 and the total number of francophone teachers is 2627, as of last year.

Nine hundred and sixteen students are currently enrolled in the B.Ed. (four-year programme) and five in the one-year programme. These include all specializations. About 20-30 students are enrolled in the secondary math major, about 20 in the secondary math minor, and about 500 in the elementary programme.

Educating Teachers of Mathematics
in the Province of Nova Scotia

Summary

	Elementary	Secondary
Preservice	3-4 Education courses ½-1 Mathematics Methods course, plus 100 hours internship. This is after a B.A. or B.Sc. of any kind. Note: Course = 2 semester, 3 hrs/week course.	3-4 Education courses, plus 1 Mathematics Methods course, plus 100 hours internship. This is after a B.A. or B.Sc. which must include a minimum of 4 mathematics courses. Note: Course = 2 semester, 3 hrs/week course.
Inservice	Essentially none.	St. Francis Xavier University has a M.A. in Teaching Mathematics which involves courses given in the summer only. Admission requirements: B.Ed. plus a major in mathematics plus "minimum averages and ranking criteria". Degree requirements: a) Three mathematics courses specifically designed for teachers. b) One Curriculum Theory course. c) One elective in Mathematics or Education or a thesis. Note: Course = 2 semester, 3 hrs/week course.

Educating Teachers of Mathematics
in the Province of Nova Scotia

General Comments

(a) Six institutions are involved in teacher training.
Eight hundred and two teachers graduated with a B.Ed. degree in the Spring of 1977.

(b) Certification procedures:

- no specialist licensing
- higher licences are obtained by taking more courses but there seems to be little guidance (or concern) about what these courses should be.

(c) Number of Teachers graduating from Nova Scotia Institutions in the Spring of 1977 (see next page).

TEACHERS GRADUATING FROM NOVA SCOTIA INSTITUTIONS

SPRING 1977

		B.Ed.			
	Senior Diploma	Part-Time	Full-Time Integrated	Full-Time Consecutive	TOTAL
Acadia University	7	-	26	142	175
Université Sainte-Anne	-	-	1	-	1
Dalhousie University	-	10	10	173	193
Mt. Allison (N.S. Students only)	-	-	-	47	47
Mt. St. Vincent Univ.	-	-	2	120	122
N.S. Teachers College	194	-	-	-	194
St. Francis Xavier Univ.	-	-	2	134	136
Saint Mary's University	-	-	-	145	145
	201	10	41	761	1013

Educating Teachers of Mathematics in the Province of Prince Edward Island

Summary

	Elementary	Secondary
Preservice	1. 4 year B.Ed. requires 4 years of university work. 2. Post degree B.Ed. requires one additional year of university work and contains 36 semester hours of course work beyond the first degree. 3. 4 year B.Ed. contains 45/126 semester hours of Education courses. 4. No mathematics course requirement but a 3 semester hour course in Mathematics Education is included.	1. 4 year B.Ed. requires 4 years of university work. 2. Post degree B.Ed. requires one additional year of university work and contains 36 semester hours of course work beyond the first degree. 3. 4 year B.Ed. contains 42/126 semester hours of Education courses. 4. Mathematics specialists require 42 semester hours in Mathematics and 3 semester hours in Mathematics Education.
Inservice	1. The U.P.E.I. Mathematics Educator from the Faculty of Education is frequently involved in inservice workshops. 2. Some school units have their own Mathematics Coordinator who conducts workshops. 3. At times the teachers conduct their own workshops.	1. The U.P.E.I. Mathematics Department is frequently involved in inservice workshops. 2. Some school units have their own Mathematics Coordinator who conducts workshops. 3. At times the teachers conduct their own workshops.

Educating Teachers of Mathematics in the Province of Prince Edward Island

General Comments

As indicated on the previous page, there are two education programmes at the University of P.E.I.:

(a) a four-year B.Ed. programme wherein students develop two majors--one in Education and one in an academic subject area. Each major represents from 42-45 semester hours of course work.

(b) a one-year (post-degree) programme.

Within each programme a student elects to take courses that prepare him/her for teaching at either the elementary or secondary level. For teaching at the elementary level, a student may choose any major's programme offered by the university. Students who wish to teach at the secondary school level, however, must major in a subject taught in most Canadian high schools.

The secondary level students seem well prepared for their role--with a well-developed major and a course in the teaching of math at the secondary level. However, we are not at all happy with the preparation of students who teach math at the elementary level. In many cases, students go out to the schools to teach math with no more background in the subject than their high school courses.

Part of the problem is associated with our degree structure. That is, the student's academic major (generally not math) makes up one-third of the courses in the degree. The education major makes up another third of the degree. The student is then left with only one-third of the degree for background courses in other subject areas. Part of the problem is also associated with the schools. As long as elementary teachers are required to teach five or more school subjects to the same class, they are bound to have deficiencies in one or more of these areas. It would seem to me that in such basic areas as language and mathematics, the schools should be using more specialist teachers. However, the fact is that most elementary schools don't use them. The result is that many children absorb anxieties about mathematics from their teachers. Most elementary teachers lack any clear idea of the broad general structure and background of mathematics.

The following are figures on the number of teachers and pupils at both the elementary and secondary levels in Prince Edward Island.

Children attending schools in P.E.I. Grades 1-12

Unit I	3,833
Unit II	6,325
Unit III	11,062
Unit IV	5,678
Unit V	684
Vocational Schools	777
	28,359

Number of Schools

71 (including two vocational schools)

Number of Teachers Listed with the P.E.I. Teachers' Federation

A total of 1,459 teachers are listed. However, some of these are teachers who are working in Board offices, etc.

Educating Teachers of Mathematics
in the Province of Newfoundland

Summary

	Elementary	Secondary
Preservice	8 semesters for B.A.(Ed.): 20 non-Education courses 20 Education courses Required minimum: 2 Mathematics courses 2 Mathematics Education courses (1 semester courses)	10 semesters for B.Ed./B.A., or B.Ed./B.Sc.: Minimum - 12 Mathematics courses Minimum - 2 Mathematics Education courses Minimum - 12 Education courses (1 semester courses)
Inservice	The summer offerings under the jurisdiction of the Division of part-time credit studies are mainly inservice. A description is attached. Non-credit courses for Inservice are provided by the Newfoundland Teachers' Association by way of Short Courses, by the Faculty of Education, by way of workshops, and by board supervisors who conduct various mini-courses to orient teachers to new programmes.	

Educating Teachers of Mathematics
in the Province of Newfoundland

General Comments

The Division of Part-time Studies at Memorial University administers the On-Campus Evening Programme, the Off-Campus Programme, the Correspondence Programme, and Summer Session.

The On-Campus Evening Programme-like many evening programmes offered by other Canadian universities--offers courses for students attending university on a part-time basis. This semester approximately sixty courses are offered and the enrolment is 1300 students.

In the Off-Campus Programme, approximately 1200 students are registered in thirty-seven centres scattered throughout Newfoundland and Labrador. Part-time and full-time instructors teach some of the courses; other courses are taught by television. These courses are produced by Memorial's Educational Television Centre, dubbed on video-tape and forwarded to centres where students meet as a class and view the video-taped lectures. Exams and assignments are written in these centres and forwarded to the University where they are corrected and returned to the students.

Eleven courses are offered by correspondence with an enrolment of 440 students.

Through Summer Session in St. John's, Corner Brook, and Grand Falls, Memorial University attempts to complement the evening, off-campus, and correspondence programmes. In other words, the Summer Programme concentrates on required degree courses and specialized Summer institutes. This gives students the opportunity to register for courses which cannot be offered--for many reasons--through the other programmes.